

Score:

Name:

## Sample Final Exam Calculus 1 - MAT 175

*Instructions: This exam should be taken without text or notes or electronic devices. Show your work, and indicate answers clearly. Cross out all work that you do not want to be graded. You should have time to check your answers carefully.*

1. (5pts.) Compute the derivative  $\frac{dy}{dx}$  for  $y = \sqrt{5} + \frac{x^5}{5} - 5 \ln x$ .
2. (5pts.) Compute the derivative  $p'(y)$  of the function  $p(y) = \pi \cos y + \frac{\pi}{y} + y$ .
3. (5pts.) Write down an equation of the tangent line to the graph of  $y = 1 + 2e^x$  at the point where  $x = 0$ .
4. (5pts.) Determine the slope of the tangent line to the graph of the equation  $x^2 + 2y^2 = 22$  at the point  $(2, -3)$ .
5. (5pts.) Compute the derivative  $P'(\theta)$  of the function  $P(\theta) = \theta \cos \theta$ .
6. (5pts.) Compute the derivative  $\frac{dw}{dz}$  of the function  $w = 3\sqrt{1 + z^5}$ .

7. (5pts.) Find the limit:

$$\lim_{\theta \rightarrow 0} \frac{3 \sin \theta}{2\theta} =$$

8. (5pts.) Find the limit:

$$\lim_{t \rightarrow \infty} \frac{\sin t}{e^t} =$$

9. (5pts.) Find the limit:

$$\lim_{x \rightarrow \infty} \frac{x^2 - 1}{x^2 + 1} =$$

10. (10pts.) Show that the derivative of  $f(x) = 3x^2 + 1$  is  $f'(x) = 6x$  by using the definition of the derivative as the limit of a difference quotient.
11. (5pts.) If the volume  $V = s^3$  of an expanding cube is increasing at the constant rate of 120 cubic inches per second, how fast is the length  $s$  of the sides increasing when the volume is 8 cubic inches?
12. (10pts.) Find where the graph of  $y = x^3 - 6x^2$  is concave up and concave down, and find all inflection points.
13. (5pts.) If the position of an object dropped from a height of 64 feet is given by  $h(t) = 64 - 16t^2$  after  $t$  seconds, find both the velocity and the acceleration of the object when it hits the ground.
14. (5pts.) For which constant  $k$  is the following function  $Q(x)$  continuous for all  $x$ ? Justify your answer.

$$Q(x) = \begin{cases} x^2 + k & \text{if } x \leq 0 \\ \cos x & \text{if } 0 < x \end{cases}$$

15. (10pts.) Find the absolute maximum and minimum values of  $f(x) = 4x^3 - 3x^2$  on the closed interval  $[-1, 1]$ .
16. (10pts.) Find all relative extrema of  $F(x) = x^4 - 4x^2 + 2$ .