Midterm Examination I

MAT175 Section B402

October 4th, 2012. 9:00AM–10:40AM

Instructions: (1) Print your name on the exam booklet. This exam is closed-book and closed-note. You cannot use any calculator for this exam. You are not allowed to talk to other students. To receive full scores, write all details explicitly. Answers without justifications and/or calculation steps may receive no score.
(2) You can use any theorem without proof if it had been proved during this course, and unless you are explicitly asked to prove the theorem — But state clearly and precisely what you are using without proof, as a part of justification.
(3) Do any 10 problems. If you did more than 10 problems, cross out solutions you wish to drop.

1. (1) Draw the graph of the following function. What is the domain?(5 points)

$$f(x) = \frac{|x - 2012|}{x - 2012}$$

(2) Find the limit $\lim_{x \to 2012} f(x)$ if exists, or explain why the limit does not exist.(5 points)

2. The $\epsilon - \delta$ definition of limit is the following: "A real-valued function f defined on an interval I, except possibly at $a \in I$, is said to have a limit L as x approaches to $a \in \mathbb{R}$ if for every $\epsilon > 0$ there exists $\delta > 0$ such that $0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$." Now let $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x$. By using the given definition of limit, prove that $f(x) \to 2$ as $x \to 1$. What if f is defined only on $\mathbb{R} - \{1\}$? Using the given definition, argue whether or not the limit exists in this case.(10 points)

3. Evaluate the following limit: (10 points)

$$\lim_{x \to 2} \frac{x^5 - 32}{x - 2}.$$

4. Evaluate the following limit: (10 points)

$$\lim_{x \to 4} \frac{\sqrt{x + 5} - 3}{x - 4}.$$

5. Evaluate the following limit: (10 points)

$$\lim_{\theta \to 0} \frac{2013 \sin \theta}{2012\theta}.$$

6. Evaluate the following limit: (10 points)

$$\lim_{x \to 0} \frac{1 - e^{-x}}{e^x - 1}.$$

7. (1) Draw the graph of the following function.(5 points)

$$f(x) = -\sqrt{4 - x^2}$$

(2) Discuss continuity of this function on the set of all real numbers.(5 points)

8. Discuss continuity of the the following function on the set of all real numbers.(10 points)

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

9. Determine whether a function $f(x) = x^5 - x^4 + x^3 - x^2 + 2x - 1$ has a zero in a closed interval $[-1, 1]$.(10 points)

10. Find all vertical asymptotes of the following function.(10 points)

$$f(x) = \frac{x^2}{x^2 - 4}$$

11. Evaluate the following limit:(10 points)

$$\lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

12. Evaluate the following limit:(10 points)

$$\lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta^2}$$

13. Find the constant a such that the function is continuous on the entire real number line.(10 points)

$$f(x) = \begin{cases} x^3 & \text{if } x \leq 2 \\ ax^2 & \text{if } x > 2 \end{cases}$$