Midterm Examination II

MAT175 Section B402

November 13th, 2012. 9:00AM–10:40AM

Instructions: (1) Print your name on the exam booklet. This exam is closed-book and closed-note. You cannot use any calculator for this exam. You are not allowed to talk to other students. To receive full scores, write all details explicitly. Answers without justifications and/or calculation steps may receive no score.
(2) You can use any theorem without proof if it had been proved during this course, and unless you are explicitly asked to prove the theorem — But state clearly and precisely what you are using without proof, as a part of justification.
(3) Do any 10 problems. 10 points each. If you did more than 10 problems, cross out solutions you wish to drop. Please do not do more than 10 problems. If you did more than 10 problems, scores might be summed over from the lowest, up to 10 problems.

1. Prove the following:

$$\frac{d}{d\theta}\tan\theta = \sec^2\theta$$

One may use $(\sin\theta)' = \cos\theta$, $(\cos\theta)' = -\sin\theta$ and the quotient rule of differentiation without proof.

2. Prove the following by using the definition of the derivative(i.e. use the limit of difference quotient):

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

3. Determine the values of x, if any, at which the graph of the following function has a horizontal tangent line:

$$f(x) = \frac{1}{3}x^3 - 2x + 7$$

[4.–5.] Find the equation of the tangent line to the graph of each function at specified w or z:

4. $f(w) = 6w - 5\ln w \quad \text{at } w = 1$

5. $y = \frac{1}{2}e^z - 3\sin z \quad \text{at } z = \pi$

6. Find constants a and b such that the function is differentiable on the entire real number line.

$$f(x) = \begin{cases} ax^3 & \text{if } x \leq 2 \\ x^2 + b & \text{if } x > 2 \end{cases}$$

7. Find the derivative:

$$(1) \quad f(\alpha) = \frac{5\alpha - 2}{\alpha^2 + 1} \qquad\qquad (2) \quad y = (3\beta^3 + 4\beta)(e^{2\beta} + 1)(\ln 7\beta^2 + 1)$$

It is enough to provide terms after taking all necessary derivatives. There will be no deduction of points about simplifying terms.

[8.–9.] Suppose a particle in motion on the real line follows the following position function:

$$s(t) = -t^2 + 4t + 2$$

8. Find the velocity function and the acceleration function for this particle.

9. Draw the position-time graph and velocity-time graph. Exhibit all intercepts which make sense.

10. By differentiating implicitly, find the equation of the tangent line to the graph of $3e^{xy} - x = 0$ at $(3, 0)$.

11. Find the derivative of $y = \arcsin x$ (or, equivalently $y = \mathrm{Sin}^{-1} x$) defined on an open interval $(-1, 1)$.

12. Find the derivative:

$$(1) \quad f(x) = \sin(\cos x) \qquad\qquad (2) \quad y = \ln \sqrt[3]{\frac{x-2}{x+2}}$$

13. The formula for the volume of a cone is the following:

$$V = \frac{1}{3}\pi r^2 h$$

Find the rate of change of the volume if dr/dt is 2 inches per minutes and $h = 3r$ when $r = 6$ inches.

14. The radius r of a circle is increasing at a rate of 3 meter per second. Find the rate of change of the area when $r = 6$ meters.

15. Prove that if f a real-valued function on an open interval I is differentiable at $a \in I$, then f is continuous at a.

16. Is the converse of the statement in Problem 15 true? i.e. is f always differentiable at $a \in I$ if f is continuous at a? Prove it or give a counterexample to disprove.