

Optional Midterm Examination

MAT175 Section B402

November 30th, 2012. 10:00AM–11:40AM(At Gillet 319)

Instructions: (1) Print your name on the exam booklet. This exam is closed-book and closed-note. You cannot use any calculator for this exam. You are not allowed to talk to other students. To receive full scores, write all details explicitly. Answers without justifications and/or calculation steps may receive no score.

(2) You can use any theorem without proof if it had been proved during this course, and unless you are explicitly asked to prove the theorem — But state clearly and precisely what you are using without proof, as a part of justification.

(3) Total 100 points. There is no extra problem.

1. Evaluate the following limit: (5 points)

$$\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 5x + 6}.$$

2. Evaluate the following limit: (5 points)

$$\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}.$$

3. Evaluate the following limit: (5 points)

$$\lim_{x \rightarrow 0} \frac{2 \tan^2 x}{x}.$$

4. Evaluate the following limit: (5 points)

$$\lim_{\phi \rightarrow \pi} \phi \sec \phi.$$

5. Evaluate the following limit: (5 points)

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{2x^2}.$$

6. Evaluate the following limit: (5 points)

$$\lim_{x \rightarrow \ln 2} \frac{e^{3x} - 8}{e^{2x} - 4}.$$

7. Find all vertical asymptotes of the following: (10 points)

$$f(t) = \frac{t^2}{\cos t - 1}$$

8. Find the constant a such that the function is continuous on the entire real number line. (10 points)

$$f(x) = \begin{cases} \frac{x^2 - a^2}{x - a} & \text{if } x \neq a \\ 8 & \text{if } x = a \end{cases}$$

9. Prove the following by using the definition of the derivative: (10 points)

$$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2}x^{-1/2}$$

10. Find the derivative: (10 points)

$$(1) \quad f(x) = \sqrt{x} - 6\sqrt[3]{x} \qquad (2) \quad y = \frac{2}{\sqrt[3]{x}} + 5 \cos x$$

11. Determine the point(s), if any, at which the graph of the following function has a horizontal tangent line: (10 points)

$$f(x) = x + 4e^x$$

12. Find dy/dx : (10 points)

$$(1) \quad xe^y - 10x + 3y = 0 \qquad (2) \quad \frac{x + \sin x}{y - \cos y} = 1$$

13. A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate 1 foot per second. When the radius is 4 feet, at what rate is the total area A of the disturbed water changing? (10 points)