Review Sheet — Optional Midterm

MAT175 Section B402

November 23, 2012

Outline of this exam: There will be 8 problems(50 points) from Chapter 2(Sections 2.2–2.5) and 5 problems(50 points) from Chapter 3(Sections 3.1–3.7). There will be no extra problems in this exam, and problems are as simple as the uniform final exam. If any of problems in this exam is not as simple as that uniform exam, the idea is provided below.

Limits and Continuity

We have seen several techniques to evaluate limit when it is given as an indeterminate form. Make sure you can find limits by accurately carrying out (1) factorization(p.82–83) (2) rationalizing(p.84) and (3) using the theorem we proved(p.85 Theorem 2.9. Examples on p.86). Not only these listed pages, do more practices using the problem set(p.87–89). I told you how to prove

$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

during the class, and actually gave you two ideas. At that time, I used an idea manipulating angle x. Now let's do this again using another simple idea: multiply $1+\cos x$ both on the numerator and the denominator.

$$\begin{aligned}\lim_{x\to 0}\frac{1-\cos x}{x} &= \lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{x(1+\cos x)} = \lim_{x\to 0}\frac{1-\cos^2 x}{x(1+\cos x)} = \lim_{x\to 0}\frac{\sin^2 x}{x(1+\cos x)}\\ &= \lim_{x\to 0}\frac{x\sin^2 x}{x^2(1+\cos x)} = \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 \lim_{x\to 0}\frac{x}{(1+\cos x)} = \left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2\left(\frac{0}{(1+\cos 0)}\right)\\ &= 1\cdot\frac{0}{(1+1)} = 1\cdot 0 = 0.\end{aligned}$$

Note that $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ is *not* equal to 0. You can calculate then what it is by employing the same idea I described in the above. I hope I would not encounter any claim that is trying to argue that $\lim_{x\to 0}\frac{1-\cos x}{x^2}=0$ from your exam booklet.

From precalculus, you should definitely have learned the following factorization:

$$a^n-b^n=(a-b)(a^{n-1}b^0+a^{n-2}b^1+\ldots+a^1b^{n-2}+a^0b^{n-1})$$

for all $n\in\mathbb{N}$. Write down formulae for $n=2,3$. Factorize the following: $e^{3x}-216$.

I have been constantly asking questions of the type of exercises 63–66 in p.100, but still I do not feel that all of you mastered questions of this type. I will ask this again — Be prepared!

We discussed the idea of Theorem 2.14 in p.105. Roughly speaking, as the denominator goes to zero as $x\to a$ where the numerator stays as a nonzero value at $x=a$, then the quotient tends to either positive or negative infinity as $x\to a$. Make sure you understand the Example 2 below the statement. I find the exercise 31 in p.108 from your homework very nice. By Theorem 2.14, this function $s(t)$ has a vertical asymptote at each of zero of the denominator. What are zeros of $\sin t$? If you do not know this, make sure looking it up from your precalculus textbook. How about zeros of $\cos x - 1$? Now the crux is that, at $t = 0$, the fraction $s(t)$ becomes an indeterminate type $0/0$. Does $s(t)$ tend to infinity as $t \to 0$? The answer is no, since $\lim_{t\to 0} \frac{t}{\sin t} = 1$. Can you work out details for the same question where $s(t) = \frac{t^2}{1-\cos t}$? How about $s(t) = \frac{t}{1-\cos t}$?

Differentiations

Since you learned what differentiation is, you should know the definition of derivative of a function at a point. If that is still confusing, go to p.119, read and understand the definition, and do some calculations: Example 3,4,5 in page 120–121. Note in particular that Example 4 is asking you to find the slope of a tangent line to the graph of the function $f(x) = \sqrt{x}$. Let me add the following question to that: explicitly write the equation of the tangent line for each of the slope you found.

We also studied various differentiation rules. These are scattered in sections 3.2–3.6, and you should master all those computational techniques. Certainly there will be questions asking you to differentiate a given function, and for that you should know differentiation rules and how to use them correctly. In particular, clearly understand the chain rule and the implicit differentiation.

From the second midterm exam, most of you did not do well about the question on related rates. You should be able to do those examples(in particular 2,3,4) in section 3.7, and homework problems. There will be a question from this section.