#1

$$\lim_{x\to 3}\frac{x^2-x-6}{x^2-5x+6} = \lim_{x\to 3}\frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x-2)} = 5.$$

#2.

$$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3}\frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{x-3\qquad(\sqrt{x+1}+2)} = \lim_{x\to 3}\frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+1}+2)} = \frac{1}{4}.$$

#3.

$$\lim_{x\to 0}\frac{2\tan^2 x}{x} = \lim_{x\to 0}\frac{2\sin^2 x}{x^2}\,\frac{x}{\cos^2 x} = \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 \lim_{x\to 0}\frac{2x}{\cos^2 x} = 0.$$

#4.

$$\lim_{\phi\to\pi}\phi\sec\phi = \lim_{t\to 0}(t+\pi)\frac{1}{\cos(t+\pi)} = -\pi.$$

let $t=\phi-\pi$. As $\phi\to\pi$, $t\to 0$. $\phi = t+\pi$

(or direct substitution: $\pi\sec\pi = \pi\frac{1}{\cos\pi} = -\pi$.) (this is enough)

#5.

$$\lim_{x\to 0}\frac{\cos x-1}{2x^2} = \lim_{x\to 0}\frac{(\cos x-1)(\cos x+1)}{2x^2\quad(\cos x+1)} = \lim_{x\to 0}\frac{-\sin^2 x}{2x^2(\cos x+1)} = \lim_{x\to 0}\frac{\sin^2 x}{x^2}\lim_{x\to 0}\frac{-1}{2(\cos x+1)} = -\frac{1}{4}.$$

#6.

$$\lim_{x\to\ln 2}\frac{e^{3x}-8}{e^{2x}-4} = \lim_{x\to\ln 2}\frac{\cancel{(e^x-2)}(e^{2x}+2e^x+4)}{\cancel{(e^x-2)}(e^x+2)} = \frac{\underbrace{e^{2\ln 2}}_{4}+2\underbrace{e^{\ln 2}}_{2}+4}{\underbrace{e^{\ln 2}}_{2}+2} = \frac{10}{4} = \frac{5}{2}.$$

#7. Since $\displaystyle\lim_{t\to 0}\frac{t^2}{\cos t-1} = \frac{1}{\lim_{t\to 0}\frac{\cos t-1}{t^2}} = \frac{1}{-\frac{1}{2}} = -2$ There is no vertical asymptote at $t=0$.
f has a removable discontinuity at $t=0$. For all other t, each of which makes $\cos t-1$ vanishing
i.e. $t=\frac{k\pi}{2}$ where $k\in\{2n+1 : n\in\mathbb{Z}\}$, f has vertical asymptotes.

#8. To make $f(x)$ continuous $\lim_{x\to a} f(x) = f(a)$. In this case the left and the right limit of f at $x=a$ agrees, and hence one only needs to check that the limit equals to $f(a)$.
i.e. $\displaystyle\lim_{x\to a}\frac{x^2-a^2}{x-a} = \underbrace{2a = 8}_{} = f(a)$ $\longrightarrow$ $\boxed{a=4.}$

#9.

$$\frac{d}{dx}(\sqrt{x}) = \lim_{\Delta x\to 0}\frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x} = \lim_{\Delta x\to 0}\frac{\cancel{x}+\cancel{\Delta x}-\cancel{x}}{\cancel{\Delta x}(\sqrt{x+\Delta x}+\sqrt{x})} = \lim_{\Delta x\to 0}\frac{1}{\sqrt{x+\Delta x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}.$$

#10. (1) $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-\frac{2}{3}} = \dfrac{1}{2\sqrt{x}} - \dfrac{2}{\sqrt[3]{x^2}}$. (2) $y' = -\frac{2}{3}x^{-\frac{4}{3}} - 5\sin x = -\dfrac{2}{3x\sqrt[3]{x}} - 5\sin x.$

#11. $f'(x) = 1+4e^x = 0$ Note that, for any $x\in\mathbb{R}$, $e^x \geq 0$. Hence $f'(x) \geq 0$

For x satisfying this the slope of the tangent line is zero (i.e. horizontal)

Thus f cannot have a horizontal tangent line.

#12. (1) $\frac{d}{dx}(xe^y - 10x + 3y) = 0$

$e^y + xe^y y' - 10 + 3y' = 0.$

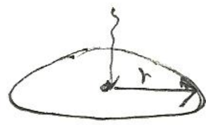
$y' = \dfrac{dy}{dx} = \dfrac{10-e^y}{3+xe^y}.$

(2) $\frac{d}{dx}(x+\sin x) = \frac{d}{dx}(y-\cos y)$

$\hookrightarrow$ $1+\cos x = (1+\sin y)\,y'$

$y' = \dfrac{dy}{dx} = \dfrac{1+\cos x}{1+\sin y}$

#13.



$\to$ 1 ft/sec $= \dfrac{dr}{dt}$

Area $A(r) = \pi r^2$ $\quad \dfrac{dA}{dt}$ at $r = 4$ ft?

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} = 2\pi\cdot 4\text{ ft}\cdot 1\text{ ft/sec} = 8\pi\text{ ft}^2/\text{sec}.$$