MAT 175
Calculus I - Practice Final Exam Problems

1. Compute the derivative of each function:

   a) $f(x) = e^{\sin x}$

   b) $g(t) = t \ln t$

   c) $h(x) = x^5 - 3x^4 + \pi x^3 + \sqrt{2}$

   d) $u(t) = \sqrt{t} + \frac{1}{t}$

   e) $v(x) = \ln(1 + x^2)$

2. By writing down an equation or sketching a graph, give an example of a differentiable function whose derivative is *never* equal to zero.

   Then give an example of a differentiable function whose derivative is *always* equal to zero.

3. Write down an equation of the tangent line to the graph of the function $F(x) = 1 - e^x$ at the point $(1, 1 - e)$.

4. Write down an equation of the tangent line to the graph of the function $G(x) = x + \ln x$ at the point $(e, e + 1)$.

5. At which points are the tangent lines to the graph of $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 6$ horizontal?

6. Compute the limits:

   a) $\lim_{x \to 2} x^2 + 3x - 1 =$

   b) $\lim_{x \to 2} \frac{x^2+3x-1}{x+1} =$

   c)$\lim_{x \to 2} \frac{x^2-4}{x-2} =$

   d) $\lim_{\theta \to 0} \frac{\sin 2\theta}{3\theta} =$

7. For which constant k is the following function $R(x)$ continuous for all x? Justify your answer.

$$R(x) = \begin{cases} \frac{x^2-4}{x+2} & \text{if } x \neq -2 \\ k & \text{if } x = -2 \end{cases}$$

8. For which x is the following function $S(x)$ continuous? Justify your answer.

$$S(x) = \begin{cases} x^2 & \text{if } x < 0 \\ \cos x & \text{if } 0 \leq x \end{cases}$$

9. Differentiate each function:

a) $u(x) = (3x)(\sin x)(e^x)$

b) $v(x) = \cos(\sin(e^x))$

10. Find the absolute maximum and minimum values of $f(x) = 3x^4 - 4x^2$ on the closed interval $[-1, 2]$.

11. Find all relative extrema of $F(x) = -3x^5 + 5x^3$.

12. Find concavity and inflection points of the graph of $y = x^4 - 4x^3$.

13. Determine the slope of the tangent line to the graph of the equation $x^2 + 4y^2 = 4$ at the point $(\sqrt{2}, -1/\sqrt{2})$. HINT: Use implicit differentiation.

14. If $G(s)$ is a differentiable function, circle the correct expression for the slope of the tangent line to the graph of $G(s)$ at the point $(0, G(0))$:

$$\lim_{\Delta s \to 1} \frac{G(\Delta s) - G(0)}{\Delta s} \qquad \lim_{\Delta s \to 0} \frac{G(1 + \Delta s) - G(1)}{\Delta s} \qquad \lim_{\Delta s \to 0} \frac{G(\Delta s) - G(0)}{\Delta s} \qquad \lim_{\Delta s \to 1} \frac{G(0 + \Delta s) - G(0)}{\Delta s}$$

15. If the area $A = \pi r^2$ enclosed by an expanding circle is increasing at the constant rate of 24π square inches per second, how fast is the radius r of the circle increasing when the area is 16π?

16. If the length s of the sides of an expanding cube is increasing at the constant rate of 2 inches per second, how fast is the volume $V = s^3$ of the cube increasing when $V = 27$?

17. Find the limits:

a) $\lim_{x\to\infty} \frac{2x+5}{3x^2+1} =$

b) $\lim_{x\to\infty} \frac{2x^2+5}{3x^2+1} =$

c) $\lim_{x\to\infty} \frac{2x^3+5}{3x^2+1} =$

d) $\lim_{x\to\infty} \frac{x+\sqrt{x}}{\sqrt{x}+99} =$

e) $\lim_{x\to\infty} \frac{1+\sin x}{e^x} =$

f) $\lim_{x\to\infty} \frac{-6x^3+2x}{2x^3+3x^2} =$

18. If the position at time t of an object moving in a straight line is given by $s = \sqrt{t}$, find the velocity and acceleration at $t = 4$.