Review Sheet — Midterm I

MAT175 Section B402

September 28, 2012

As already noted, the best preparation for any exam in this course will be a through understanding of textbook materials. Most importantly, try to understand examples, homework problems and the quiz problems and practice several times until you can do them by your own understanding. Repeat the same training having a day or two of interval to actually solidify your understanding. Some of exam problems will be identical to examples, homework problems and quiz problems. Put emphasis on them while you are preparing for the exam.

Section 2.2

In this section, it is important to understand the concept of limit. We could see what the limit of a function is by using graphs. The notion of limit, as we have discussed, had nothing to do with the fact whether the function is defined at the point where the limit is evaluated. Exercise 11–20 in p.75 are some sample questions to see if this point is very clear to you. While looking at your homework, most of you knew the answer for #13 correctly, but the explanations were not satisfactory. Say, most of you did not get this problem. I want you to make this very clear before we move on. You can look at p.70 example 3, or use the notion of side limits to give a correct argument for non-existence. We also spent quite a lot of time for understanding a formal definition of limit, and you will see the first question in the quiz was essentially the same question to that of example 6 and 7 in p.73. We shall not go into very deep about this technique, however, I want you to remember the definition as it is appearing in the blue-box in p.72 and know how to use it in very simple cases.

Section 2.3

This section is technical core in what we have been studying about limits. Every theorems, examples and homework problems are important. In particular, each of examples in p.83–84 and in p.86 are in a very nice form to be asked. Thoroughly understand these examples, and work out similar problems in p.88 starting from reviewing homework problems. In case of p.86 examples and related problems, mostly, you will need to use any of facts that was established in theorem 2.9 in p.85. Unless explicitly asked, you do not need to prove them while you are using, but you should remember them precisely, as **these will not be given in the exam**. Also the second one in the theorem 2.9, we discussed in class, and I gave you two ideas. For the third one, this is the definition of e, and we discussed in class about how to prove $\lim_{x\to 0}(e^x - 1)/x = 1$. Recall that you could substitute $t = e^x - 1$. Then you will end up with a form that you need to use this definition of e. In p.88, exercise 79, 80 are in a good form to ask if you know this limit and how to use it. Also calculations appearing in exercise 61–64 in the same page are important in the next chapter. If you feel difficulties about these, that's probably because you are not certain about how to deal with those terms in the numerator. Look up your precalculus textbook if necessary. If you are not familiar with Δx, tell me, for example, what's the difference between $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$ and $(x+\Delta x)^3 = x^3+3x^2\Delta x+3x\Delta x^2+\Delta x^3$. Also make sure you can do the quiz problems.

Section 2.4

First of all, make sure you can distinguish the existence of the limit and the notion of continuity. Exercises 1–6 in p.98 are good for checking pictorially if you understand it correctly. The next a bunch of exercises 7–28 are basically asking if you know the definition of side limits(p.92–93) which is important. Now exercises 37–60 are questions to ask, simultaneously, if you know the above two points. Also work out Exercises 63–66 in p.100. There was a similar problem asked in the sample final exam of the department. We discussed the intermediate value theorem(p.97), and the example in p.98 is a very typical one. You can practice more through exercises 83–86. A question: what if I asked whether the function $h(x)$ has a zero in a closed interval $[-\pi/2, \pi/2]$? Note that, even if $h(-\pi/2)$ and $h(\pi/2)$ are both positive, still there exists $x = \pm\pi/4$ such that $h(\pm\pi/4) = 0$. When you use the intermediate value theorem, **all conditions should be mentioned explicitly**. For example, without the following quote: "Since h is a continuous function on a closed interval $[0, \pi/2]$" if you used the intermediate value theorem to solve #85, a large portion of score will be deducted even if your other arguments are correct. Also have a careful look at examples in page 96. The third one, for example, is excellent, because I can ask the squeeze theorem through this question.

Section 2.5

More comments will be made when we are discussing this section on Tuesday, October 2nd. However, in this section, if you know how to characterize infinite limit(definition blue-box in p.103), know how to draw graphs at least of the form in example 1 in p.104 without using any graphic utility, and if you understand what theorem 2.14 in p.105 means, then that's almost it. Know as well what vertical asymptote is and when it appears. If you understand theorem 2.14, that will clarify this point.