Review Sheet — Midterm II

MAT175 Section B402

October 29, 2012

The same and equally important statement from the previous review sheet: "As already noted, the best preparation for any exam in this course will be a through understanding of textbook materials. Most importantly, try to understand examples, homework problems and the quiz problems and practice several times until you can do them by your own understanding. Repeat the same training having a day or two of interval to actually solidify your understanding. Some of exam problems will be identical to examples, homework problems and quiz problems. Put emphasis on them while you are preparing for the exam."

How to read this review sheet? Read it sentence by sentence. If it mentions any page number and/or problem number, go to that page and that problem, and see if you are comfortable with it. If you are, go to the next sentence. If you are not, study that part until you become very comfortable. In this way, go through all sentences.

Section 3.1

In this section, we studied about what differentiation is. Recall that I mentioned that there are two ways you can bring the notion of slope from precalculus, which you are already familiar with, to the case of a graph of a function which is not necessarily linear: (1) pick two points, say P and Q from the graph and draw a straight line joining them (2) pick a point and draw a tangent line. Now, by taking limit $Q \to P$, we could make sense the slope of a *tangent* line as a limit of the slope of the straight line $\overline{PQ}$ if the limit exists. More precisely, we could define what tangent line is, in this way. Now the definition(blue-box) in p.117 and in p.119 are formalizing this concept, and the slope of the tangent line at a point was called the *derivative* of a function at that point. If this is too abstract, I have a concise and nice example for you: Example 2 in p.118. This is certainly one of the most elementary form, but is extremely important. Add the following question as well: *What is the equation of the tangent line at* $(-1, 2)$*?* A good habit whenever you encountered a question like this is to try to write down the following on your worksheet, before doing anything:

$$\left\{ \begin{array}{c} \text{point: } (-1,2) \\ \text{slope: ???} \end{array} \right. \Rightarrow \text{line: ???}$$

Another very important technique was actually using the definition of the type as in p.119 to calculate the derivative of a function, and this technique were used over and over again. Examples 3,4 and 5 in page 120 and 121 will give you a good practice. Mimic the solution while practicing, if you are not sure about these calculations. See also exercises 21–24 in page 124. Try all these and check with the answer you can get by using differentiation rules.

Among what is conceptually important, I explained that differentiability is a stronger condition than continuity, and even proofs were very simple. Study theorem 3.1 in p.123. Remember the proof also. The converse of this is not true, and there are many examples: see example 6 and 7 in page 122. Now you might ask the following: Is the derivative of a differentiable function always continuous? i.e. is $f'(x)$ always continuous at $x = a$, granted $f(x)$ is differentiable at $x = a$? The answer is in page 126, Exercise 103. Study the function g.

Section 3.2, 3.3 and 3.4

In these sections, we proved differentiation rules, and have seen lots of examples. See if you memorized them all:

- $c' = 0$ if c is a constant.
- $(x^r)' = rx^{r-1}$ where r is a rational number.
- $[af(x) \pm bg(x)]' = af'(x) \pm bg'(x)$ where a, b are constants.
- $(fg)' = f'g + fg'$ where f, g are functions.
- $(f/g)' = (f'g - fg')/g^2$

And there were another box of formula I gave you:

- $(\sin x)' = \cos x$
- $(\cos x)' = -\sin x$
- $(e^x)' = e^x$
- $(\ln x) = 1/x$

For practicing proofs, I recommend the last four formula for your study. Mastering these technique will help not only your understanding of this chapter but of the previous chapter as well. If you can prove all four equalities in theorem 3.10 in page 144 by yourself, that will be a wonderful practice for the rest formula as well — Believe me, this is easy! In sections 3.2 and 3.3, **all examples and homework problems are important**. If you still want me to pick some problems, instead, try to solve any homework problem in these sections, by making random choices. If you look at the sample final exam, you will see there are many questions you can attempt and easily get the answer by mastering the above list of formula. So please try to do as many as you can.

What important in these sections is understanding one-dimensional motions. Example 11 in p.135, Example 10 in p.146 and Exercise 93, 95 in page 138 are problems you can try. If you understand those problems (7.–10.) in Quiz 2, you will quickly notice that actual math questions in these physics-like problems are in fact easy.

About the chain rule, it is fine if you do not remember the proof, but you should be a master in doing calculation by using it! Again in this section, all examples and homework problems are important, and recall that by using the differentiation for ln function as in the form theorem 3.13 in p.157, we could carry out calculations as in example 15 in p.158 easily. I told you that by remembering some precalculus facts, all the formula in theorem 3.15 in 159 boil down to a set of simple exercises you can do by using those formula I listed above — this means, you don't need to memorize these as formula. The following were precalculus facts I mentioned in class:

$$a^{\log_b c} = c^{\log_b a} \qquad\qquad \log_b a = \frac{\log_c a}{\log_c b}$$

If you still don't remember these, memorize right now!

Section 3.5 and 3.6

Now having chain rule at hand, if we are given an implicit form of a function $F(x, y) = 0$, we could calculate dy/dx without changing $F(x, y) = 0$ into an explicit form. The idea was regarding y as a function of x, and hence we multiplied y' after differentiating any term with respect to y, as we are usually doing so whenever we apply the chain rule: whenever a differentiation is done with respect to a function, we multiplied the function after differentiating it. First see if you can do Example 2 in p.167 and Example 4 in p.168. If you could, practice with exercises 1–20 in p.171 and 25–34 in p.172. Try also Example 7 in page 170 and Example 9(a powerful technique!!) in p.171. Do some more exercises from Exercises 65–74 in page 173. Understand the function arcsin by studying example 6 in p.169.

We also studied the inverse function theorem, and one of morals of the theorem was that we even don't need to find the inverse function to calculate the derivative of the inverse. I will draw a cartoon for you to elucidate what this means more clearly. In fact, all you need to understand is:

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}.$$

Here x in the LHS is what you will regard as a inverse function: $x = g(y)$ where g is the inverse of $y = f(x)$. Let's sketch a proof of one of formula in theorem 3.18 in p.177. For example, if you try to find the derivative of $\arctan x$, first put $y = \arctan x = \mathrm{Tan}^{-1}x$. You want to find dy/dx By definition of the inverse function,

$$y = \mathrm{Tan}^{-1}x \qquad \Leftrightarrow \qquad \tan y = x.$$

From this, you can easily compute dx/dy or even dy/dx directly by using implicit differentiation. Complete this sketch to get the answer as in the book, and try the case of arccos.

Section 3.7

If you understand how to apply the chain rule appropriately, all the difficulties are coming from setting up the right equation to reflect the reality well — sometimes this can be quite complicated. For our purpose, it would be safe if you can understand Examples 1,2 and 3 from p.182–184, and if you can, try examples 4,5 and 6 as well. Make sure you have the following precisely when you are starting: (1) an equation (2) a given rate of change(s) and (3) a rate of change you want to calculate. We will discuss these examples carefully in class.