Precalculus Assessment Quiz

MAT175 Section B402

August 28th, 2012

Instructions: Please PRINT your ID, first and last name on the yellow worksheet. Also put a sheet number on each sheet, on the upper-right corner. 10 points each, unless stated otherwise. You can leave before 10:40AM if you are done. You are not allowed to use a calculator in completing these problems.

1. Sketch each graph of the following functions:
(1) $2x + 3y = 1$ (Exhibit: x-intercept, y-intercept)
(2) $y = -x^2 - x + 1$ (Exhibit: apex, y-intercept)
(3) $y = 4^{x-3} + 3$ (Exhibit: any asymptote, y-intercept)
(4) $y = \ln(x - 5) + 2$ (Exhibit: any asymptote, x-intercept)
(5) $y = |x + 2| + 7$ (Exhibit: any point where the slope is abruptly changed, y-intercept)

2. Find a positive real number A and an angle $\alpha \in [0, \frac{\pi}{2}]$ such that $\sin x - \cos x = A \sin(x - \alpha)$.(5 points)

3. Prove that

$$\log_a b = \frac{\log_c b}{\log_c a}.$$

4. Establish an one-to-one correspondence between the set of all real numbers and a circle minus a point.

5. Prove the following identities:

(1) $\tan x + \cot x = \sec x \csc x$ (2) $\dfrac{\tan^3 x - 1}{\tan x - 1} = \tan^2 x + \tan x + 1$

6. (1) Prove the following identitity:

$$\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

(2) Write $\sin 5x \cos 3x$ into sum or difference of trigonometric functions.

7. A triangle $\triangle ABC$ is given. Let the length of side which is not abutting to the angle A(respectively B, C) be a(respectively b, c). Using your favorite method, prove that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ for some real number $R > 0$.

8. (1) What is the *law of cosines*? (2 points)
(2) The converse of the Pythagorean theorem is also true. Using the law of cosines, prove that the

converse of the Pythagorean theorem is also true. (3 points)
(3) By proving the law of cosines in (1), explain how the Pythagorean theorem itself implies its converse.(10 points)

9. When a vendor prices key chains at \$5 each, she sells 210 key chains. For each \$1 she raises the price, she sells 10 fewer key chains. USE AN EQUATION to determine what she should charge to maximize her revenue from sales.

10. The population North Oblivion is now 1400 people and is known to double every 12 years.
(1) Write a function that gives the population, $P(t)$, after t years.
(2) How many years will it take for the population to reach 4200? (A formula will suffice.)