#1. (1) $f(x) = 2x^3 + 4x^2 + 3x$ $\qquad f'(x) = 6x^2 + 8x + 3$

(2) $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$ $\qquad f'(x) = \frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{5}x^{-\frac{4}{5}} = \dfrac{1}{3\sqrt[3]{x^2}} + \dfrac{1}{5\sqrt[5]{x^4}}$.

#2. (1) $f(x) = \dfrac{x^3}{2\sin x + 1}$ $\qquad f'(x) = \dfrac{3x^2(2\sin x+1) - x^3 \cdot 2\cos x}{(2\sin x+1)^2} = \dfrac{x^2(3(2\sin x+1) - 2x\cos x)}{(2\sin x+1)^2}$

(2) $f(x) = \sqrt{x}\, e^x + \ln x^2$ $\qquad f'(x) = \frac{1}{2}x^{-\frac{1}{2}}e^x + x^{\frac{1}{2}} + \frac{2}{x} = \frac{1}{\sqrt{x}}\left(\frac{1}{2}e^x + x + \frac{2}{\sqrt{x}}\right)$

$$= \frac{1}{x}\left(\frac{\sqrt{x}\,e^x}{2} + x\sqrt{x} + 2\right)$$

#3. $\dfrac{d}{dx}\ln x = \lim\limits_{\Delta x \to 0} \dfrac{\ln(x+\Delta x) - \ln x}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{1}{\Delta x}\ln\left(1 + \dfrac{\Delta x}{x}\right) = \lim\limits_{\frac{\Delta x}{x} \to 0} \ln\left(1 + \dfrac{\Delta x}{x}\right)^{\frac{1}{\Delta x/x}} \cdot \dfrac{1}{x} = \dfrac{1}{x}$,

Since $\lim\limits_{\frac{\Delta x}{x} \to 0}\left(1 + \dfrac{\Delta x}{x}\right)^{\frac{1}{\frac{\Delta x}{x}}} = e$ and as $\Delta x \to 0$, $\dfrac{\Delta x}{x} \to 0$ since $x \neq 0$ on which $\ln$ is defined.

#4. $\dfrac{d}{d\theta}\cot\theta = \dfrac{d}{d\theta}\left(\dfrac{1}{\tan\theta}\right) = \dfrac{d}{d\theta}\left(\dfrac{\cos\theta}{\sin\theta}\right) = \dfrac{(\cos\theta)'\sin\theta - \cos\theta(\sin\theta)'}{\sin^2\theta} = \dfrac{-1}{\sin^2\theta}$

$$= -\csc^2\theta.$$

#9. As shown in #8, maximum $= s_0 + \dfrac{v_0^2}{2g}$. Of course, the velocity of the particle at the maximum is zero.

#10. At $s(t) = 0$, we obtain, by solving the quadratic equation

$s_0 + v_0 t - \frac{1}{2}gt^2 = 0$, $\quad t = \dfrac{v_0 \pm \sqrt{v_0^2 - 4\cdot\frac{1}{2}g\cdot(-s_0)}}{g} = \dfrac{v_0 \pm \sqrt{v_0^2 + 2gs_0}}{g}$.

$\Leftrightarrow \frac{1}{2}gt^2 - v_0 t - s_0 = 0$

Since $g > 0$ and $s_0 > 0$, $\sqrt{v_0^2 + 2gs_0} > v_0$, and hence the minus sign in the numerator yields negative time which we can drop. Thus we have

$$t = \frac{v_0 + \sqrt{v_0^2 + 2gs_0}}{g}.$$

#5. Let $f(x) = \cos x$, $h(x) = ax+b$. If f is differentiable at $x=0$,

then
$$\lim_{\Delta x \to 0^-} \frac{f(0+\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{h(0+\Delta x) - h(0)}{\Delta x}$$

$$\Rightarrow \lim_{\Delta x \to 0^-} \frac{\cos \Delta x - 1}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{a\Delta x}{\Delta x}$$

$$\Rightarrow \quad a = 0.$$

To be differentiable, f must be continuous. It follows that

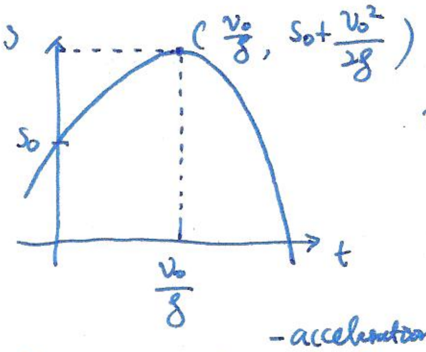
$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} h(x) \quad \Rightarrow \quad 1 = a \cdot 0 + b \quad \Leftrightarrow \quad b = 1.$$

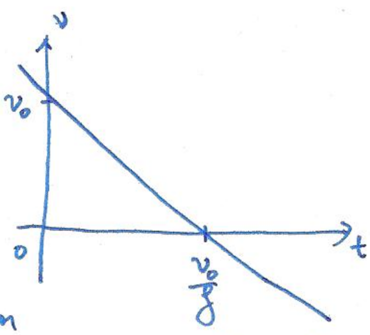
Clearly $f(x)$, $h(x)$ are differentiable on their domain minus $\{0\}$.

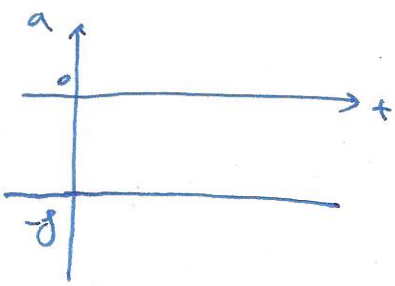
#6. $f'(x) = 9x^8 + 49x^6 + 9x^2 + 9$. Since x^8, x^6 and $x^2 \geq 0$ for all x in reals, $f'(x) \geq 9 > 8$. Thus f cannot have any tangent line that has slope 8.

#7. $S(t) = S_0 + v_0 t - \frac{1}{2} g t^2$
$v(t) = S'(t) = v_0 - gt$
$a(t) = v'(t) = -g$.

#8. Observe that
$$S(t) = S_0 + v_0 t - \tfrac{1}{2} g t^2$$
$$= -\tfrac{1}{2} g \left(t^2 - \tfrac{2v_0}{g} t\right) + S_0$$
$$= -\tfrac{1}{2} g \left(t - \tfrac{v_0}{g}\right)^2 + \tfrac{1}{2} g \cdot \left(\tfrac{v_0}{g}\right)^2 + S_0$$
$$= -\tfrac{1}{2} g \left(t - \tfrac{v_0}{g}\right)^2 + \frac{v_0^2}{2g} + S_0.$$







It is a constant ~~speed~~ -acceleration motion with the acceleration $-g$. Note that this is a motion one throws an apple vertically upward with initial velocity v_0 at the height S_0. And then by the gravity, the apple falls down at an acceleration $-g$. Here minus sign means the opposite direction to the direction of v_0 (upward).