A Solution to the Sample Final Exam

MAT104 Section F401

Spring 2013, CUNY Lehman College

Instructions: Next to each problem number, related sections in the textbook are indicated. You can firstly attempt the sample final exam on your own, without any help of others, and compare using this solution. If you could not solve a problem seamlessly or wish to practice more, go to the indicated section of the textbook, and try problems of similar type, starting from homework problems and **Part I** problems in midterm exams(solutions available on the webpage).

1.[Section 2.7]

$$\begin{aligned} & 7 - 3x \geq 31 \\ \Leftrightarrow \quad & -3x \geq 24 \\ \Leftrightarrow \quad & 3x \leq -24 \\ \Leftrightarrow \quad & x \leq -8 \end{aligned}$$

Answer: $x \leq -8$

2.[Section 3.6] First observe that

$$\begin{aligned} & 6x + 2y = 9 \\ \Leftrightarrow \quad & 2y = -6x + 9 \\ \Leftrightarrow \quad & y = -3x + \frac{9}{2}. \end{aligned}$$

Hence the given straight line has its slope -3. Recall that a straight line that is perpendicular to a straight line with slope m has its slope $m' = -\frac{1}{m}$. i.e. slopes of these two straight lines are related by the formula

$$mm' = -1.$$

Hence we are trying to find the equation of a straight line that has slope $\frac{1}{3}$ and passing through

$(3, -4)$. That is:

$$y - (-4) = \frac{1}{3}(x - 3)$$

Answer: $y = \frac{1}{3}x - 5$

3.[Section 5.3]

$$\begin{aligned}(x^2 - 4x + 7)(x + 3) &= x^3 - 4x^2 + 7x + 3x^2 - 12x + 21 \\ &= x^3 - x^2 - 5x + 21\end{aligned}$$

Answer: $x^3 - x^2 - 5x + 21$

4.[Section 5.4]

$$\begin{aligned}(-4a^{-2}b^3)^{-3}(8ab)^2 &= \frac{(8ab)^2}{(-4a^{-2}b^3)^3} = \frac{8^2a^2b^2}{(-4\left(\frac{1}{a^2}\right)b^3)^3} \\ &= \frac{8^2a^2b^2}{(-4)^3\left(\frac{1}{a^2}\right)^3 b^9} = \frac{8^2a^2b^2}{(-4)^3\left(\frac{1}{a^6}\right)b^9} \\ &= \frac{2^6a^2b^2}{-2^6\left(\frac{1}{a^6}\right)b^9} = \frac{2^6a^2b^2 \times a^6}{-2^6\left(\frac{1}{a^6}\right)b^9 \times a^6} \\ &= \frac{2^6a^8b^2}{-2^6b^9} = \frac{a^8b^2}{-b^9} \\ &= -\frac{a^8}{b^7}\end{aligned}$$

Answer: $-\frac{a^8}{b^7}$

5.[Section 5.4] Writing a decimal expression in scientific notation means writing a given number A in a form of $N \times 10^k$ where A and $N \times 10^k$ are equal, N satisfies $1 \leq N < 10$, and k is an integer. Hence $.00000000405 = 4.05 \times 10^{-9}$.

Note that

$$\begin{aligned}.00000000405 &= .0000000405 \times 10^{-1} && \text{This is not yet a scientific notation.} \\ &= .000000405 \times 10^{-2} && \text{This is not yet a scientific notation.} \\ &= .00000405 \times 10^{-3} && \text{This is not yet a scientific notation.} \\ &= .0000405 \times 10^{-4} && \text{This is not yet a scientific notation.} \\ &= .405 \times 10^{-8} && \text{This is not yet a scientific notation.} \\ &= 4.05 \times 10^{-9} && \text{This is a scientific notation,}\end{aligned}$$

since it is of a form $N \times 10^k$ with $1 \leq N < 10$.

Answer: 4.05×10^{-9} — While writing your solution to a problem of this sort, it is enough to write only the answer correctly. Of course, you don't need to exhibit what are not scientific notations. Also, you don't need to reiterate what is the definition of the term *scientific notation* unless it is explicitly asked.

6.[Section 6.1]

$$\begin{aligned} 24x^4 - 54x^8 &= 6x^4(4 - 9x^4) = 6x^4(\sqrt{2} - \sqrt{3}x)(\sqrt{2} + \sqrt{3}x)(2 + 3x^2) \\ &= 6x^4(2 - 3x^2)(2 + 3x^2) = 6x^4(\sqrt{2} - \sqrt{3}x)(\sqrt{2} + \sqrt{3}x)(\sqrt{2} + i\sqrt{3}x)(\sqrt{2} - i\sqrt{3}x) \end{aligned}$$

Answer: One of the following, depending on what the question is really asking. If the problem is about factorizations over $\mathbb{Q}$, the answer is

$$6x^4(2 - 3x^2)(2 + 3x^2).$$

If over $\mathbb{R}$, then it becomes

$$6x^4(\sqrt{2} - \sqrt{3}x)(\sqrt{2} + \sqrt{3}x)(2 + 3x^2.)$$

If over $\mathbb{C}$, it becomes

$$6x^4(\sqrt{2} - \sqrt{3}x)(\sqrt{2} + \sqrt{3}x)(\sqrt{2} + i\sqrt{3}x)(\sqrt{2} - i\sqrt{3}x).$$

7.[Section 9.2] You can use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

but in this case, it is easy to solve it directly by completing the square:

$$\begin{aligned} x^2 - 4x = 3 \quad &\Leftrightarrow \quad x^2 - 4x + 4 = 7 \quad \Leftrightarrow \quad (x + 2)^2 = 7 \\ \Leftrightarrow x + 2 = \pm\sqrt{7} \quad &\Leftrightarrow \quad x = -2 \pm \sqrt{7} \end{aligned}$$

Answer: $x = -2 \pm \sqrt{7}$

8.[Section 7.2]

$$\begin{aligned} \frac{x+6}{x^2 + x - 20} - \frac{3}{x-4} &= \frac{x+6}{(x+5)(x-4)} - \frac{3(x+5)}{(x+5)(x-4)} = \frac{x + 6 - 3(x+5)}{(x+5)(x-4)} \\ &= \frac{-2x + 6 - 15}{(x+5)(x-4)} = -\frac{2x+9}{(x+5)(x-4)} \end{aligned}$$

Answer: $-\frac{2x+9}{(x+5)(x-4)}$

9.[Section 7.2]

$$\begin{aligned}\frac{x^2+x-6}{10x^2} \div \frac{x^2-9}{2x^8} &= \frac{x^2+x-6}{10x^2} \times \frac{2x^8}{x^2-9} = \frac{(x+3)(x-2)}{10x^2} \times \frac{2x^8}{(x+3)(x-3)} \\ &= \frac{x^6(x-2)}{5(x-3)}\end{aligned}$$

Answer: $\frac{x^6(x-2)}{5(x-3)}$.

10.[Section 7.3]

$$\begin{aligned}\frac{\frac{1}{x^2}+\frac{5}{x}+6}{2x+1} &= \frac{\left(\frac{1}{x^2}+\frac{5}{x}+6\right)\times x^2}{(2x+1)\times x^2} = \frac{1+5x+6x^2}{x^2(2x+1)} \\ &= \frac{6x^2+5x+1}{x^2(2x+1)} = \frac{(2x+1)(3x+1)}{x^2(2x+1)} = \frac{3x+1}{x^2}\end{aligned}$$

Answer: $\frac{3x+1}{x^2}$

11.[Section 11.2] Recall that $\log_a a = 1$. Taking $\log_5$ both sides gives:

$$x\log_5 125 = (x+2)\log_5 25 \Leftrightarrow x\log_5 5^3 = (x+2)\log_5 5^2 \Leftrightarrow 3x = 2(x+2).$$

Hence $x = 4$.

Answer: $x = 4$

12.[Section 3.2] Since $f(x) = 3x - x^2$, by plugging-in -3 into x, we obtain $f(-3) = 3\cdot(-3)-(-3)^2 = -9-9 = -18$.

Answer: -18

13.[Section 9.6] Recall that by changing a general form $y = ax^2+bx+c$ into a normal form $y = a(x-p)^2+q$, we can obtain the vertex (p,q) of a given parabola. Hence the idea is completing the square:

$$y = 4x + x^2 = x^2+4x+4-4 = (x+2)^2-4.$$

Thus the vertex is $(-2,-4)$.

Answer: $(-2,-4)$

14.[Trigonometry] According to the statement of the problem, the length of the hypotenuse is 18 feet, and the angle between the hypotenuse and the adjacent is 65°. Recall that the hypotenuse and the opposite(height h) are related by sin:

$$0.91 = \sin 65^\circ = \frac{\text{opposite } h}{\text{hypotenus}} = \frac{h}{18}.$$

Hence $h = 18 \times 0.91 = \frac{18\times 91}{100} = \frac{1638}{100} = 16.38$

Answer: 16.38

15.[Section 11.2]

$$\frac{\log_4 8}{\log 1000} = \frac{\log_{2^2} 2^3}{\log_{10} 10^3} = \frac{\frac{\log_2 2^3}{\log_2 2^2}}{3} = \frac{\frac{3}{2}}{3} = \frac{1}{2}$$

Answer: $\frac{1}{2}$