Solutions to Midterm 2

#1. $y = 3x^2 + 6x + 10$

$= 3(x^2 + 2x) + 10$

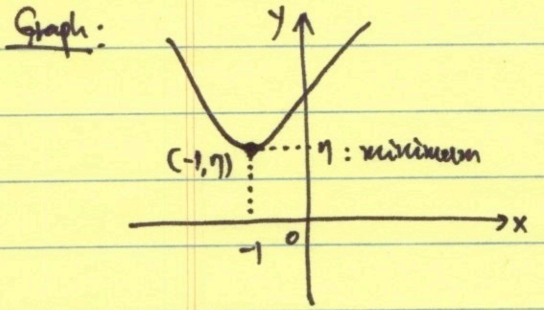
$= 3(x^2 + 2x + 1 - 1) + 10$

$= 3(x+1)^2 - 3 + 10$

$= 3(x+1)^2 + 7.$

Vertex: $(-1, 7)$

Graph:

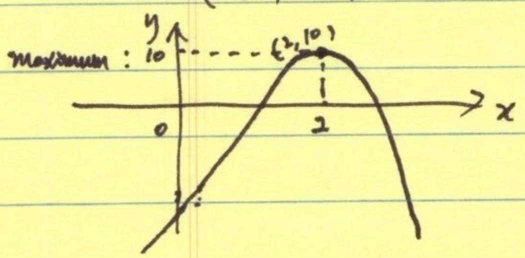


#2. $y = -2x^2 + 8x + 2$

$= -2(x^2 - 4x + 4 - 4) + 2$

$= -2(x-2)^2 + 10.$

Vertex: $(2, 10)$



#3 $y = a(x+4)^2 + 4$ with $(0,1)$

$1 = a(0+4)^2 + 4$

$-3 = 16a$

$a = -\frac{3}{16}$. So $\underline{\underline{y = -\frac{3}{16}(x+4)^2 + 4}}$

#4. $y = a(x-1)^2 - 1$ with $(0,1)$.

$1 = a(0-1)^2 - 1.$

$\Leftrightarrow a = 2.$

So $\underline{\underline{y = 2(x-1)^2 - 1}}$

#5. $f(x) = 2e^{x-1}$. Inverse of f?

Let $y = f(x)$. Inverse of f is $f^{-1}(x) = y$ below:

$x = 2e^{y-1}$

$\ln \frac{x}{2} = y - 1$

$y = \ln \frac{x}{2} + 1$. Domain of $y = f^{-1}(x)$ is $(0, \infty)$,

Since the range of $f(x) = 2e^{x-1}$ is $(0, \infty)$.

#6. $g(x) = \frac{1}{2}e^{3x}$. Inverse of g?

Let $y = g(x)$. Inverse of g is obtained by $g^{-1}(x) = y$ below:

$x = \frac{1}{2}e^{3y}$

$\ln 2x = 3y$

$y = \frac{1}{3}\ln 2x.$

Since the range of $g(x)$ is $(0, \infty)$, the domain of $g^{-1}(x)$ is $(0, \infty)$.

#7. Sales = 8000 when price = 50

Sales = 7900 when price = 51

Slope of price–Sales graph:

$\frac{7900 - 8000}{51 - 50} = -100$. So,

Sales $= -100(\text{price} - 50) + 8000.$

Let x = price.

Revenue = Sales × price

$= -100x(x-50) + 8000x$

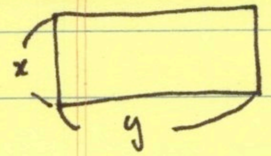
$= -100x^2 + 5000x + 8000x$

$= -100(x^2 - 130x + 65^2 - 65^2)$

$= -100(x-65)^2 + \underbrace{65^2 \cdot 100}_{=422500}$

So when (1) x = price = $\underline{\underline{65}}$, (2) the sales at max is $\underline{\underline{422{,}500}}$.

#8.

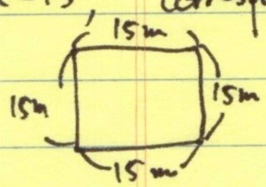


$2x+2y=60.$

$\text{Area} = xy.$

$$\begin{aligned}\text{Area} = xy &= x(30-x)\\ &= -x^2+30x\\ &= -(x^2-30x+15^2-15^2)\\ &= -\underbrace{(x-15)^2}_{\text{when } x=15} + \underbrace{15^2}_{\text{max.}}.\end{aligned}$$

Since Area is maximized when $x=15$, corresponding $y=15$.



#9 profit $P(x) = \underset{R(x)}{\text{Revenue}} - \underset{C(x)}{\text{Cost}}$

$$\begin{aligned} &= 800x-2x^2-(2x^2+1000)\\ &= -4x^2+800x-1000.\\ &= -4(x^2-200x)-1000\\ &= -4(x^2-200x+10000-10000)-1000\\ &= -4(x-100)^2+40000-1000.\\ &= -4(x-100)^2+39000.\end{aligned}$$

when $x=100$, max. profit is 39000.

#10. B_0 = initial bacteria $=100$.

(1) In 30 hours

$B(30) = B_0 \cdot 3^{\frac{30}{5}} = 100 \cdot 3^6$ bacterias are in there.

(2) $\underline{\underline{B(t) = B_0 3^{\frac{t}{5}}}}.$

(3) $1800 = 100 \cdot 3^{\frac{t}{5}}$

$18 = 3^{t/5} \Rightarrow \frac{t}{5}\log_3 3 = \log_3 18$

so $\underline{\underline{t = 5\log_3 18.}}$ //

#11. $\tilde{P}(1850) = 600.$

(1) In 2000,

$$\tilde{P}(2000) = 600 \cdot 2^{\frac{2000-1850}{25}}$$

$$\underline{\underline{= 600 \cdot 2^6}} \;//$$

(2) $\underline{P(y) = 600 \cdot 2^{\frac{y}{25}}}$ //

(3) $4000 = 600 \cdot 2^{\frac{y}{25}}$

$\frac{20}{3} = 2^{\frac{y}{25}}$

$\underline{y = 25\log_2 \frac{20}{3}}$, years after 1850, the population exceeds 4000.

#12. Initial deposit $D_0 = 1000$.

3% annual, compounded quarterly.

$$D(t) = D_0\left(\left(1+\frac{3\%}{4}\right)^4\right)^t$$

$$\underline{\underline{= 1000 \cdot \left(1+\frac{3}{400}\right)^{4t}}} \;//$$

When $2000 = D(t)$,

$2000 = 1000\left(1+\frac{3}{400}\right)^{4t}$

$\ln 2 = 4t\ln\left(1+\frac{3}{400}\right)$

$$\underline{\underline{t = \frac{4\ln(1+\frac{3}{400})}{\ln 2}.}} \;//$$