Solutions to Midterm Exam II.

#1. $\dfrac{f(x)}{g(x)} = \dfrac{x-3}{x^2-9}$. Since $x^2-9 = (x-3)(x+3) = 0$
$\Leftrightarrow x = +3$ or -3,

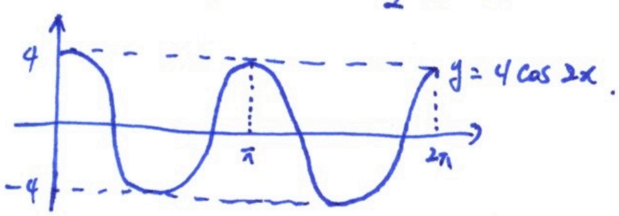
$\dfrac{f(x)}{g(x)}$ is defined on entire real line except ± 3.

At $x = \pm 3$, the denominator vanishes and hence the fraction is not defined.

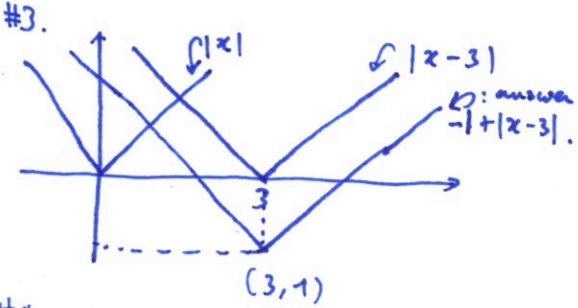
Answer: $\mathbb{R} - \{-3, 3\}$.

#2. Note that amplitude $= 4$

period $= \dfrac{2\pi}{2} = \pi$.



#3.

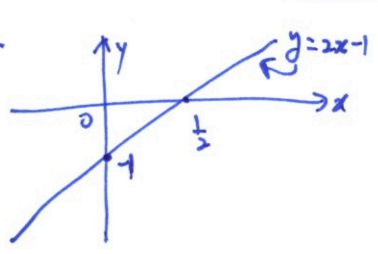


#4. Slope: 2 (perpendicular to $y = -\frac{1}{2}x + 1$)
point $(1,1)$.

$\Rightarrow$ point-slope formula

$y - 1 = 2(x-1)$. So $\underline{y = 2x - 1}$.

Sketch



#5. Vertex formula

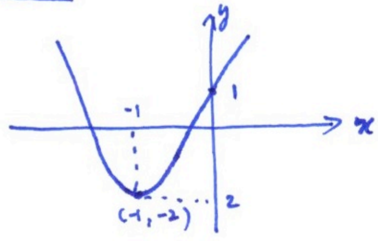
x coordinate of the vertex $= -\dfrac{b}{2a} = -\dfrac{6}{2\cdot 3} = -1$.

$y\big|_{x=-1} = 3\cdot(-1)^2 + 6\cdot(-1) + 1$
$= 3 - 6 + 1 = -2$.

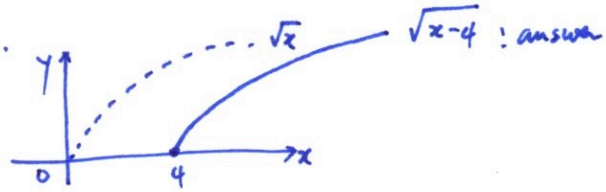
So the vertex $= (-1, -2)$

y-intercept $= 1$.

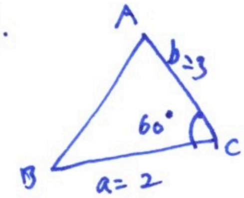
Sketch



#6.



#7.



$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$: Cosine law

$c^2 = 4 + 9 - 2\cdot 2\cdot 3\cdot \frac{1}{2} = 4 + 9 - 6 = 7$

So $\underline{c = \sqrt{7}}$.

#8. The line passes through

$(0,3)$
$(4,0)$. Slope $= \dfrac{0-3}{4-0} = -\dfrac{3}{4}$.

point: $(0,3)$.

point-slope formula $\Rightarrow$ $y - 3 = -\frac{3}{4}(x - 0)$

$\boxed{y = -\frac{3}{4}x + 3}$

#9 vertex $(2,3)$

normal form

$y = a(x-2)^2 + 3$

passing through $(0,0)$

$\Rightarrow 0 = a(0-2)^2 + 3$

$a = -\dfrac{3}{4}$

So $\boxed{y = -\frac{3}{4}(x-2)^2 + 3}$

#10. $f(x) \overset{\text{let}}{=} y = 500e^{.01x}$

Inverse function: $x = 500e^{.01y}$

$\Leftrightarrow x = 500e^{\frac{1}{100}y}$

$\Leftrightarrow \frac{x}{500} = e^{\frac{1}{100}y}$.

Take ln $\Leftrightarrow$ $\ln\frac{x}{500} = \frac{1}{100}y$

$$\boxed{y = 100\ln\frac{x}{500}}$$

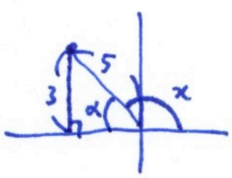
Domain: $\{x \mid x > 0\}$.

#11. $\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)+1-(3x+1)}{h}$

$= \frac{3x+3h+1-3x-1}{h}$

$= \frac{3h}{h} = \underline{\underline{3}}$.

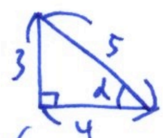
#12.



In Quadrant II

sin +
cos −
tan −

$\sin x = +\sin\alpha$
$\cos x = -\cos\alpha$
$\tan x = -\tan\alpha$



$\tan\alpha = \frac{3}{4}$.

Hence $\tan x = -\tan\alpha = -\frac{3}{4}$.

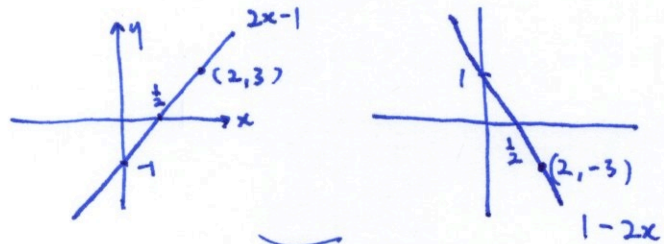
#13. $\sin(a+b) = \sin a\cos b + \cos a\sin b$

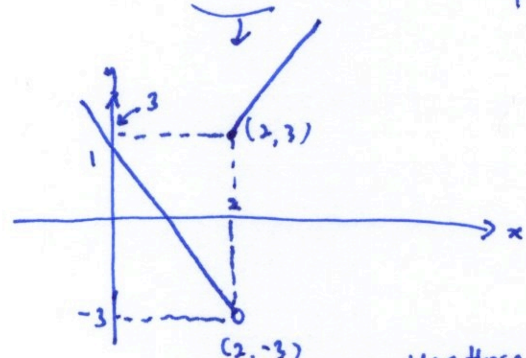
$\sin(a+\frac{\pi}{2}) = \sin a\underbrace{\cancel{\cos\frac{\pi}{2}}}_{0} + \cos a\underbrace{\sin\frac{\pi}{2}}_{=1}$.

$= \cos a$.

#15.

$F(x) = \begin{cases} 2x-1 & x \geq 2 \\ 1-2x & x < 2 \end{cases}$





#16.

0 Year	200 Shares	\$75/Share
1 Year	205 Shares	\$70/share

Use these to compute "Values in Yr"

Use these to compute "Shares" in Yr

Share in yr $= S = \frac{205-200}{1-0}(Y-0)+200$

$= 5Y + 200$

Value in yr $= V = \frac{70-75}{1-0}(Y-0)+75$

$= -5Y + 75$

Total Value $= S\cdot V = (5Y+200)(-5Y+75)$

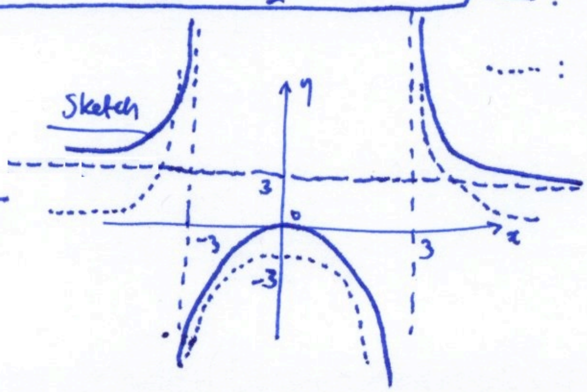
$= -25Y^2 - 1000Y + 375Y + 15000$

$= -25Y^2 - 625Y + 15000$.

Vertex formula $-\frac{b}{2a} = \frac{625}{-50} = -\frac{25}{2}$ (cont'd)

#14. $y = \frac{3x^2}{x^2-9} = \frac{3x^2-27+27}{x^2-9} = \frac{3(x^2-9)}{x^2-9} + \frac{27}{x^2-9}$

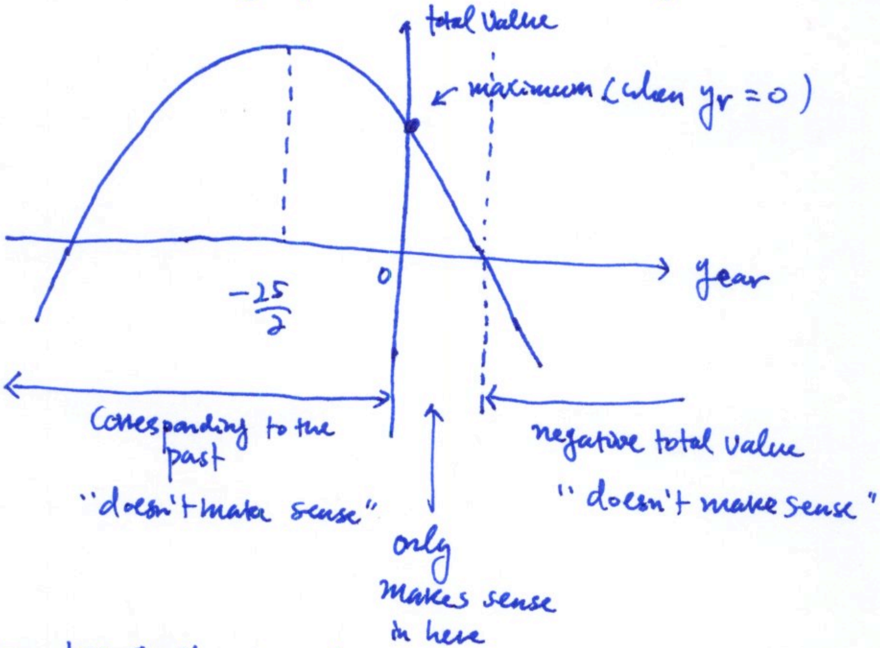
$= \underline{\underline{3 + \frac{27}{x^2-9}}}$



——— : $3 + \frac{27}{x^2-9}$

······ : $\frac{27}{x^2-9}$

So the x-coordinate of the vertex is negative. The graph will be then, roughly,



From this graph, we observe that, to maximize the total value, Mary should sell all shares at this point (year = 0) to get maximum value \$15000.

#17. $J_0 = 60\,g$ (initial)

(1) 12 days = 3 days × 4.

So $J(12\text{ days}) = 60 \cdot \left(\frac{1}{2}\right)^4$.

(2) $J(t) = J_0\left(\frac{1}{2}\right)^{\frac{t}{3}} = 60 \cdot \left(\frac{1}{2}\right)^{\frac{t}{3}}$.

(3) $60-50 = 60\left(\frac{1}{2}\right)^{\frac{t}{3}}$

$\frac{10}{60} = \left(\frac{1}{2}\right)^{\frac{t}{3}}$

$-\ln 6 = \ln\frac{1}{6} = -\frac{t}{3}\ln 2$

$t = \frac{3\ln 6}{\ln 2}$.

You can Solve either given #16 in the Sheet or this #16 by making your own choice.

#16. When NY Yankees sells tickets at $5 each, they sell 300 tickets. For each $1 they raise the price, they sell 10 fewer tickets. Use an equation to determine what Yankees should charge to maximize their revenue. (Don't need to calculate the maximum revenue. Just answer "at which price")

Sol: (price, Sales)

$(5 , 300)$

$(6 , 290)$ $\longrightarrow$ slope $= \dfrac{290 - 300}{6 - 5} = -10.$

Point - Slope formula $\boxed{y - y_0 = m(x - x_0)}$

Let Sales $= s$

price $= p$

$$s - 300 = -10(p - 5)$$

$$s = -10p + 50 + 300$$

$$\Leftrightarrow s = -10p + 350.$$

$$\text{Revenue} = \text{Sales} \times \text{Price} = s \cdot p = (-10p + 350)p$$

$$= -10p^2 + 350p$$

① by Completing the Square

$$= -10(p^2 - 35p + 17.5^2 - 17.5^2)$$

$$= -10(p - 17.5)^2 + \underbrace{10 \cdot 17.5^2}_{\text{maximum revenue}}$$

when $p = 17.5$ revenue is maximized

or ② Using the vertex formula,

$a = -10$

$b = 350$

when $p = -\dfrac{b}{2a}$

$$= -\frac{350}{-20} = \underline{\underline{17.5}}.$$