Lesson 2: Solution to an exercise

Fall 2013 MAT175 Section C401[19514]

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Exercise. Show that the slope of a straight line that is perpendicular straightline having its slope m is given by $-\frac{1}{m}$, when m is a nonzero real number.

Proof. Suppose l_1 and l_2 be perpendicular straight lines in $\mathbb{R}^2$ with slopes m_1 and m_2, respectively. We also assume that $m_1 > m_2$ and m_1, m_2 are nonzero finite numbers. It suffices to prove that each of l_1 and l_2 has its y-intercept zero. That is,

$$l_1: \quad y = m_1 x, \qquad\qquad l_2: \quad y = m_2 x.$$

Let $P = (1, m_1)$ and $Q = (1, m_2)$. Then $\triangle OQP$ is a right-angled triangle with $\angle O = 90°$. Hence by Pytagorean theorem,

$$\overline{OQ}^2 + \overline{OP}^2 = \overline{QP}^2,$$

which is $(1^2 + m_2^2) + (1^2 + m_1^2) = (m_1 - m_2)^2$, and thus it follows that $m_1 m_2 = -1$. □

Another Proof. Given any straightline l with the slope m, let θ be the angle between x-axis and l where $-90° < \theta < 90°$. Then $m = \tan\theta$, by definition of the slope(and tangent). Any straightline l' that is perpendicular to l and the x-axis form an angle $\theta + 90°$, and the slope m' of l' is $\tan(\theta + 90°) = -\cot\theta$. Hence $mm' = -1$. □