

Score:

Name:

Sample Final Exam MAT 175

Instructions: This exam should be taken without text or notes or electronic devices. Show your work, and indicate answers clearly. Cross out all work that you do not want to be graded. You should have time to check your answers carefully.

- (5pts.) Compute the derivative $\frac{dy}{dx}$ for $y = \sqrt{3} + \frac{3}{x^2} + \sqrt{x}$.
- (5pts.) Compute the derivative $r'(t)$ of the function $r(t) = \frac{\pi}{t} + \frac{t}{\pi} - \pi t$.
- (5pts.) Write down an equation of the tangent line to the graph of $y = x + \sin x$ at the point where $x = 0$.
- (5pts.) Determine the slope of the tangent line to the graph of the equation $\frac{x^2}{2} + y^3 = 9$ at the point $(-4, 1)$.
HINT: Implicit differentiation.
- (5pts.) Compute the derivative $\frac{dz}{dx}$ of the function $z = x^2 e^x$.
- (5pts.) Compute the derivative $P'(\theta)$ of the function $P(\theta) = \ln(2 + \sin \theta)$.
- (5pts.) Find the limit:

$$\lim_{t \rightarrow \infty} \frac{\sin t}{1 - e^t} =$$

- (5pts.) Find the limit:

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} =$$

- (5pts.) Find the limit:

$$\lim_{x \rightarrow 0} \frac{x^2 + 10}{9x^3 + 1} =$$

- (5pts.) If the area $A = s^2$ of an expanding square is increasing at the constant rate of 3 square inches per second, how fast is the length s of the sides increasing when the area is 49 square inches?
- (10pts.) Find where the graph of $y = -x^3 + 6x^2 + 12$ is concave up and concave down, and find all inflection points. (HINT: $y' = -3x^2 + 12x$ and $y'' = -6x + 12$.)
- (5pts.) If the position of a particle moving in a straight line is given by $x(t) = t^3 + 3t$ after t seconds, find both the velocity and the acceleration of the particle when $t = 5$.
- (10pts.) Find the absolute maximum and minimum values of $g(x) = 2x^3 - x^2$ on the closed interval $[0, 1]$. (HINT: $g'(x) = 6x^2 - 2x$.)
- (10pts.) Find all relative extrema of $F(x) = x + \frac{1}{x}$. (HINT: $F'(x) = 1 - \frac{1}{x^2}$.)
- (10pts.) Show that the derivative of $f(x) = 3x - x^2$ is $f'(x) = 3 - 2x$ by using the definition of the derivative as the limit of a difference quotient.
- (5pts.) For which constant k is the following function $Q(x)$ continuous for all x ? Justify your answer.

$$Q(x) = \begin{cases} k - e^x & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } 0 < x \end{cases}$$