

Midterm Exam I

Fall 2013, MAT 175 Section C401[19514]

October 3rd, 2013. 11:00AM–12:40PM.

Instructions: Print your name on the exam booklet. This exam is closed-book and closed-note. You cannot use any electronic device in this exam. You are not allowed to talk to other students. Write all details explicitly. Answers without justifications and/or calculation steps may receive no score.

Part I — 5 points each, total 50 points

1.(Sample Final I-9) Evaluate the following limit:

$$\lim_{x \rightarrow 6} \frac{2x + 1}{\sqrt{x + 3}}.$$

2.(Sample Final I-9) Evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{2014x + 10}{8x^{2013} + 5}.$$

3.(Sample Final I-8) Evaluate the following limit:

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}.$$

4.(Sample Final I-8) Evaluate the following limit: (5 points)

$$\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 5x + 6}.$$

5. Evaluate the following limit:

$$\lim_{x \rightarrow 3} \frac{\sqrt{x + 1} - 2}{x - 3}.$$

6. Evaluate the following limit:

$$\lim_{x \rightarrow 1} \frac{\sqrt{2x - 1} - \sqrt{x}}{x - 1}.$$

7.(Sample Final II-7 variant) Evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}.$$

8.(Sample Final II-7 variant) Evaluate the following limit:

$$\lim_{x \rightarrow 1} \frac{2 \tan(x - 1)}{7(x - 1)}.$$

9. (Sample Final I-16) Find the constant a such that the function is continuous on the entire real number line.

$$f(x) = \begin{cases} \frac{x^2 - a^2}{x - a} & \text{if } x \neq a \\ 16 & \text{if } x = a \end{cases}$$

10. (Sample Final I-16) Find the constant a such that the function is continuous on the entire real number line.

$$f(x) = \begin{cases} x^3 & \text{if } x \leq 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

Part II — points varies, total 50 points

11. (5 points) Evaluate the following limit: (*Hint*: $\cos \pi = -1$)

$$\lim_{\phi \rightarrow \pi} \phi \cos \phi.$$

12. (5 points) Evaluate the following limit:

$$\lim_{x \rightarrow e} (\ln x^2 + 2^{x/e})$$

13. (5 points) Let $f(x) = x^2$. Calculate:

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

14. (5 points) Find all vertical asymptotes of the following:

$$f(x) = \frac{x+1}{x^2-1}.$$

15. (10 points) By using the squeeze theorem (i.e. the sandwich lemma), show that:

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0.$$

16. (10 points) For the function $f(x) = \frac{|x|}{x}$, discuss existence of limit as x approaches to 0.

17. (10 points) Using the intermediate value theorem, show that $f(x) = x^3 - 3x$ has at least one zero in the interval $[1, 4]$. (No point will be given if you do not use the intermediate value theorem)