Midterm Exam I Solution

#1. $\lim_{x\to 6} \frac{2x+1}{\sqrt{x+3}} = \frac{13}{\sqrt{9}} = \underline{\underline{\frac{13}{3}}}$ //

#2. $\lim_{x\to 0} \frac{2014x+10}{8x^{2013}+5} = \frac{2014\cdot 0+10}{8\cdot 0^{2013}+5} = \frac{10}{5} = \underline{\underline{2}}$ //

#3. $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \underline{4}$ //

#4. $\lim_{x\to 3} \frac{x^2-x-6}{x^2-5x+6} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x-2)} = \underline{5}$ //

#5. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)}$

$= \lim_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+1}+2)} = \underline{\underline{\frac{1}{4}}}$ //

#6. $\lim_{x\to 1} \frac{\sqrt{2x-1}-\sqrt{x}}{x-1} = \lim_{x\to 1} \frac{(\sqrt{2x-1}-\sqrt{x})(\sqrt{2x-1}+\sqrt{x})}{(x-1)(\sqrt{2x-1}+\sqrt{x})}$

$= \lim_{x\to 1} \frac{2x-1-x}{(x-1)(\sqrt{2x-1}+\sqrt{x})}$

$= \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{2x-1}+\sqrt{x})} = \underline{\underline{\frac{1}{2}}}$ //

#7. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \frac{\frac{\sin 2x}{2x}\cdot 2\cancel{x}}{\frac{\sin 3x}{3x}\cdot 3\cancel{x}}$

as $x\to 0$, $2x\to 0$ and $3x\to 0$ $\Rightarrow$ $= \frac{\lim_{2x\to 0} \frac{\sin 2x}{2x}\cdot 2}{\lim_{3x\to 0}\frac{\sin 3x}{3x}\cdot 3} = \underline{\underline{\frac{2}{3}}}$ //.

#8. Let $x-1=\theta$. As $x\to 1$, $\theta\to 0$.

So $\lim_{\theta\to 0} \frac{2\tan\theta}{7\theta} = \lim_{\theta\to 0} \frac{2}{7\cos\theta}\cdot\frac{\sin\theta}{\theta} = \underline{\underline{\frac{2}{7}}}$ //

#9. $\lim_{x\to a} \frac{x^2-a^2}{x-a} \;=\; 16$ (To be continuous this equality must hold.)

$\lim_{x\to a} \frac{x^2-a^2}{x-a} = \lim_{x\to a} f(x)$, $16 = f(a)$

$\Rightarrow \lim_{x\to a} \frac{x^2-a^2}{x-a} = \lim_{x\to a} x+a = 2a = 16.$

Thus $\underline{\underline{a = 8}}$ //

#10. To be continuous $\lim_{x\to 1} f(x)$ and $f(1)$ must agree; To calculate $\lim_{x\to 1} f(x)$, we need to have its existence first, and the existence of limit depends on the parameter a:

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} ax^2 = a$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^3 = 1$.

So the limit exists only if $a=1$.

Now if $a=1$, $\lim_{x\to 1} f(x) = 1 = f(1)$. Hence $\underline{a=1}$ makes $f(x)$ continuous on $\mathbb{R}$.

#11. $\lim_{\phi\to\pi} \phi\cos\phi = \pi\cdot(-1) = \underline{\underline{-\pi}}$.

#12. $\lim_{x\to e} \ln x^2 + 2^{\frac{x}{e}} = \lim_{x\to e} 2\ln x + 2^{\frac{x}{e}} = 2+2 = \underline{\underline{4}}$ //

#13. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-x^2}{h} = \lim_{h\to 0} \frac{2xh+h^2}{h}$

$= \lim_{h\to 0} 2x+h = \underline{\underline{2x}}$ //

#14. $f(x) = \frac{x+1}{(x+1)(x-1)}$. $\lim_{x\to 1^\pm} f(x) \to \pm\infty$

whereas $\lim_{x\to -1} f(x) = -\frac{1}{2}$. Hence $x=1$ is a vertical asymptote of $f(x)$.

#15. Since $-1 \le \sin\frac{1}{x} \le 1$ for all $x\in\mathbb{R}-\{0\}$ and $\lim_{x\to 0} -x = 0 = \lim_{x\to 0} x$, by the Squeeze theorem, $\lim_{x\to 0} x\sin\frac{1}{x} = 0$, since $-x \le x\sin\frac{1}{x} \le x$.

#16. $\lim_{x\to 0} f(x)$ does not exist, because

$\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{x\to 0^+}\frac{x}{x} = 1 \ne -1 = \lim_{x\to 0^-}\frac{-x}{x} = \lim_{x\to 0^-}\frac{|x|}{x}$.

i.e. left & right limit doesn't agree at $x=0$.

#17. $f(x)$ is continuous everywhere on $[1,4]$, and $f(1) = -2$ $f(2) = 8-6 = 2$.

Hence by the intermediate value theorem, $\exists\, \xi \in [1,2] \subset [1,4]$ such that $f(\xi) = 0$.