Midterm Exam II Solution.

#1. $y = \sqrt{7} + \frac{1}{x^3} + 2\sqrt{x}$

$$\frac{dy}{dx} = 0 - \frac{3}{x^4} + \frac{1}{\sqrt{x}}.$$

$$= \underline{\underline{-\frac{3}{x^4} + \frac{1}{\sqrt{x}}}} //$$

#2. $y = \underbrace{e^2 + \frac{1}{e}}_{\text{constant}} + 3e^x + 2\ln x$

$$\frac{dy}{dx} = 0 + 3e^x + \frac{2}{x}$$

$$= \underline{\underline{3e^x + \frac{2}{x}}} //$$

#3. $f(t) = \frac{x^2}{t^2} + \frac{t^2}{x^2} + tx$

$$\underline{\underline{f'(t) = -\frac{2x^2}{t^3} + \frac{2t}{x^2} + x.}} //$$

#4. $p(c) = \pi c \cos(\pi x) + \frac{x}{c} + c + ce^x$

$$p'(c) = \underline{\underline{\pi \cos(\pi x) - \frac{x}{c^2} + 1 + e^x}} //$$

#5. point: $(0, y|_{x=0} = 1)$.

Slope: $y'|_{x=0} = 2e^{2x} + 2|_{x=0} = 4.$

$\Rightarrow y - 1 = 4(x - 0)$

$\Leftrightarrow \underline{\underline{y = 4x + 1}} //$

#6. point: $(0, y|_{x=0} = 1)$

Slope: $y'|_{x=0} = 1 - \sin x|_{x=0} = 1.$

$\Rightarrow y - 1 = 1 \cdot (x - 0)$

$\Leftrightarrow \underline{\underline{y = x + 1}} //$

#7. point: $(2,1)$

slope: $\left.\frac{dy}{dx}\right|_{(2,1)} = ?$

$\frac{dy}{dx} = ?$ $\quad 4x^2 + 9y^2 = 25$

Implicit diff. $\quad 8x\,dx + 18y\,dy = 0.$

answer

$$\frac{dy}{dx} = \frac{8x}{-18y} = \frac{-4x}{9y} \underset{\text{at}(2,1)}{=} -\frac{8}{9}.$$

So $y - 1 = -\frac{8}{9}(x - 2) = -\frac{8}{9}x + \frac{16}{9}$

$\Leftrightarrow \boxed{y = -\frac{8}{9}x + \frac{25}{9}} //$

#8. point $(\sqrt{2}, 1)$

Slope: $\left.\frac{dy}{dx}\right|_{(\sqrt{2},1)} = ?$

$\frac{dy}{dx} = ?$ $\quad x^2 - y^2 = 1$

Implicit diff. $\quad 2x\,dx - 2y\,dy = 0. \Leftrightarrow \frac{dy}{dx} = \frac{x}{y}$

$$\underset{\text{at}(\sqrt{2},1)}{=} \frac{\sqrt{2}}{1} = \sqrt{2}.$$

So $y - 1 = \sqrt{2}(x - \sqrt{2})$

answer

$\Leftrightarrow \boxed{y = \sqrt{2}x - 1} //$

#9. $z = x^3 e^{3x}$

$$\frac{dz}{dx} = 3x^2 \cdot e^{3x} + x^3 \cdot 3e^{3x}$$

$$= \underline{(3x^3 + 3x^2)\, e^{3x}.} //$$

#10. $A(\theta) = \theta e^\theta \cos\theta$

$$\frac{dA}{d\theta} = A'(\theta) = 1 \cdot e^\theta \cos\theta + \theta \cdot e^\theta \cos\theta - \theta e^\theta \sin\theta$$

$$= \underline{e^\theta(\cos\theta + \theta\cos\theta - \theta\sin\theta)} //$$

#11. $P(l) = \ln(l^2 + \sin l)$

$$P'(l) = \frac{2l + \cos l}{l^2 + \sin l}$$

#12. $Q(\pi) = \cos(\sin(\pi^2))$

$$Q'(\pi) = -\sin(\sin(\pi^2))\left(\sin(\pi^2)\right)'$$
$$= -\sin(\sin(\pi^2))\cos(\pi^2)\cdot 2\pi.$$
$$= -2\pi\sin(\sin(\pi^2))\cos(\pi^2)$$

#13. $h(t) = 64 - \frac{1}{2}gt^2$

$h'(t) = -gt$

$h''(t) = -g$ ... $\boxed{\text{acceleration}}$ (constant at anytime)

When the object hits the ground,

$h(t)$ is zero.

$0 = 64 - \frac{1}{2}gt^2.$

$t^2 = \frac{128}{g}$. Hence when

$t_{\text{ground}} \overset{\text{let}}{=} t = 8\sqrt{\frac{2}{g}}$, the object hits the ground.

Hence the velocity at t_{ground} is

$$v'(t_{\text{ground}}) = -g \cdot 8\sqrt{\frac{2}{g}} = -8\sqrt{2g}.$$ ... $\boxed{\text{velocity}}$

#14. $x(t) = t^4 + 2t$

$v(t) = x'(t) = 4t^3 + 2 \overset{\text{at } t=1}{\Longrightarrow} 6$

$a(t) = x''(t) = 12t^2. \overset{\text{at } t=1}{\Longrightarrow} 12$

#15. $A(a) = \frac{\sqrt{3}}{4}a^2$

$\frac{dA}{dt} = 3$ inch^2/sec : given.

Find: $\frac{da}{dt}$.

Chain rule:

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2}a\frac{da}{dt} \quad \cdots (*)$$

Another given condition says

when Area $= 4\sqrt{3}$, we're supposed to find $\frac{da}{dt}$.

From $A = 4\sqrt{3} = \frac{\sqrt{3}}{4}a^2$,

$a = 4$.

Hence from (*), $\frac{da}{dt} = \frac{dA/dt}{\frac{\sqrt{3}}{2}a}$

$$= \frac{3}{\frac{\sqrt{3}}{2}\cdot 4}$$

$$= \frac{\sqrt{3}}{2}$$

#16. $V(a) = \frac{\sqrt{2}}{12}a^3$.

Given that $\frac{dV}{dt} = 120$ inch^3/sec.

When $V(a) = \frac{8\sqrt{2}}{12} = \frac{\sqrt{2}}{12}a^3$

$\Leftrightarrow a^3 = 8$

So $a = 2$.

Chain rule:

$$\frac{dV}{dt} = \frac{\sqrt{2}}{4}a^2\frac{da}{dt} = 120.$$

$$\frac{da}{dt} = \frac{120}{\frac{\sqrt{2}}{4}\cdot 2^2} = \frac{120}{\sqrt{2}} = 60\sqrt{2}$$

#17. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2(x+h)^2 - 1 - (2x^2 - 1)}{h}$

$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 1 - 2x^2 + 1}{h}$

$= \lim_{h \to 0} \frac{4x\cancel{h} + 2h^{\cancel{2}}}{\cancel{h}} = 4x.$

#18. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$

$= \lim_{h \to 0} \frac{2x\cancel{h} + h^{\cancel{2}} + \cancel{h}}{\cancel{h}} = 2x + 1.$