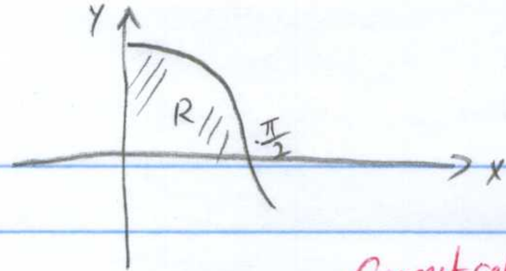
MATH 155-08 Exam I Solution

#1.



$$V = \int_0^{\frac{\pi}{2}} \pi y^2 \, dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \pi \int_0^{\frac{\pi}{2}} \frac{1}{2}(1+\cos 2x)\, dx$$

$$= \frac{\pi}{2}\left[x + \frac{1}{2}\sin 2x \right]_0^{\frac{\pi}{2}} = \frac{\pi}{2}\left(\frac{\pi}{2} + \frac{1}{2}\cdot 0\right) = \underline{\left(\frac{\pi}{2}\right)^2}.$$

Correct setup (+3)
Correct integral (+3)
~~correct answer~~ (+2)
Correct procedure (+2)

#2.

$$L = \int_0^1 \sqrt{1+(y')^2}\, dx \qquad y' = \frac{1}{3}\cdot\frac{3}{2}(x^2+2)^{\frac{1}{2}}\cdot 2x$$

$$= \int_0^1 \sqrt{1+x^2(x^2+2)}\, dx \qquad = x(x^2+2)^{1/2}$$

$$= \int_0^1 \sqrt{(x^2+1)^2}\, dx$$

$$= \int_0^1 (x^2+1)\, dx = \frac{1}{3}x^3 + x \Big|_0^1 = \frac{1}{3}+1 = \underline{\frac{4}{3}}$$

Correct setup (+3)
y' or $(y')^2$ (+2)

#3.

$$A = \int_0^5 2\pi y \sqrt{1+(y')^2}\, dx \qquad y' = \frac{1}{2}(4x+6)^{-\frac{1}{2}}\cdot 4$$

$$= \int_0^5 2\pi\sqrt{4x+6}\sqrt{1+4(4x+6)^{-1}}\, dx \qquad = 2(4x+6)^{-\frac{1}{2}}.$$

$$= \int_0^5 2\pi\sqrt{4x+6+4}\, dx \quad (*) \qquad {}^{30}_{10} \; t = 4x+10 \; {}^{5}_{0}$$

$$dt = 4dx$$

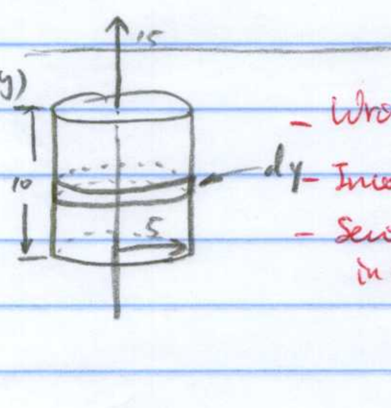
$$= \frac{1}{4}\int_{10}^{30} 2\pi\sqrt{t}\, dt$$

$$= \frac{1}{\cancel{4}}\cancel{2}\pi \cdot \frac{\cancel{2}}{3} t^{3/2}\Big|_{10}^{30} = \frac{1}{3}\pi\, 30\sqrt{30} - \frac{1}{3}\pi\, 10\sqrt{10})$$

$$= \underline{\frac{1}{3}\pi\cdot 10\sqrt{10}\,(3\sqrt{3}-1)}.$$

Correct Setup (+3)
up to (*) (+2)
Complete answer (+5)

#4



$$W = \int dW = \int mg(15-y) = \int \rho\cdot 25\pi\, dy \cdot g\cdot(15-y)$$

$$= 25\pi\rho g \int_0^{10} 15-y \; dy$$

$$= 25\pi\rho g \left(15y - \frac{1}{2}y^2\Big|_0^{10}\right)$$

$$= 25\pi\rho g\,(150-50)$$

$$= \underline{2500\pi\rho g}$$

- Writing what F is (+3)
- Incompleteness (−2)
- Serious mistakes in setting up the integral (−4) to no pts.

#5. $f(x) = -x^2+8$

• No partial credit.

Let $y = f(x)$

$$\left.\frac{dx}{dy}\right|_{(7,1)} = \frac{1}{\left.\frac{dy}{dx}\right|_{(1,7)}} = \frac{1}{\left.-2x\right|_{x=1}} = -\frac{1}{2}$$

#6. $$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x}\,dx = -\int_0^{\frac{\pi}{2}} \frac{-\sin x}{1+\cos x}\,dx = -\ln|1+\cos x|\Big|_0^{\frac{\pi}{2}}$$

$$= -\ln 1 - (-\ln 2) = \ln 2$$

Sign/ Substitution mistake (−2)

incompleteness (−2)

#7. $$\int \frac{1}{16x^2+1}\,dx = \int \frac{1}{(4x)^2+1}\cdot d\left(\frac{1}{4}\cdot 4x\right) = \frac{1}{4}\int \frac{1}{(4x)^2+1}\,d(4x)$$

$$= \frac{1}{4}\tan^{-1}(4x) + C.$$

• Formula for $\int \frac{1}{1+x^2}dx$ (+2)

• Substitution mistake (−4)

#8. $x^x = e^{x\ln x}$. (*)

• up to (*) : (+4)

$$\lim_{x\to 0^+} x\ln x = \overbrace{\lim_{x\to 0^+} \frac{\ln x}{1/x}}^{\infty/\infty\text{-form}} \underset{\text{L'Hôpital}}{=} \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} -x = 0.$$

Hence $\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln x} = e^0 = 1$.

#9. $$\int \frac{\sin x + \tan x}{\cos^2 x}\,dx = \int \frac{\sin x}{\cos^2 x}\,dx + \int \frac{\sin x/\cos x}{\cos^2 x}\,dx$$

• No partial credit except minor mistakes

$$= \int \frac{-1}{\cos^2 x}\,d(\cos x) + \int \frac{1}{\cos^3 x} - d(\cos x)$$

Correct integral each (+4)

$$= \frac{1}{\cos x} + \frac{1}{2}\,\frac{1}{\cos^2 x} + C$$

• integration const. missing (−2) ~~Obtained by int. by parts~~

#10. Let $I = \int e^{2x}\cos x\,dx = e^{2x}\sin x - 2\int e^{2x}\sin x\,dx = e^{2x}\sin x - 2\left(-e^{2x}\cos x + 2\int e^{2x}\cos x\,dx\right)$

$$= e^{2x}\sin x + 2e^{2x}\cos x - 4I$$

$$I = \frac{1}{5}\left(e^{2x}\sin x + 2e^{2x}\cos x\right) + C$$

• minor mistake (−2)

• Completeness (+2)

• correct integration by parts each (+4)