REVIEW PROBLEMS

1. Evaluate the following integral:
$$\int \sin^3 x \, dx$$

2. Prove that the area of an ellipse whose equation is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $ab\pi$.

3. Evaluate
$$\int \frac{dx}{(1+x^2)^{3/2}}.$$

4. Evaluate
$$\int \frac{dx}{x^2 - 10x + 24}.$$

5. Find the constant k that satisfies the following equation:
$$\int_{-\infty}^{\infty} \frac{k}{1+9x^2} dx = 1.$$

6. (5 points each) Let $f(x) = \frac{1}{x^p}$, where $0 < p < \infty$. Discuss the convergence of the definite integral $\int_1^\infty f(x) dx$ in the following cases:

(1) When $0 < p < 1$, (2) When $p = 1$, and (3) When $p > 1$.

7. Find the value that the following infinite sum converges to:
$$\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)}.$$

8. Find the limit of the sequence as $n \to \infty$:
$$a_n = \frac{\cos n}{n}.$$

9. Evaluate the following geometric series: $1 + \frac{2}{7} + \frac{2^2}{7^2} + \ldots + \frac{2^n}{7^n} + \ldots$.

10. (5 points) Evaluate $\int_1^2 \ln x \, dx$. (Hint: Integration by parts.)