Review Problems for Exam III
MATH 155 Section 08
Exam date and time: December 17th, 2015. 6:20PM–8:20PM

Review Problems

1. (5 points) Let R be the region in the xy-plane bounded by the curves $y = \sqrt{x}$ and $y = x^3$. Set up an integral which equals the volume of the solid formed by rotating the region R around the x-axis. Do not evaluate the integral.

2. (10 points) Find the surface area of an ellipsoid obtained by rotating an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about x-axis, where $a > b$.

3. (10 points) Calculate the following integral:

$$\int e^{2x} \cos 3x dx.$$

4. (5 points each) Evaluate or show divergence: (1) $\int_1^\infty \frac{1}{x^2} dx$ (2) $\int_0^\infty e^{-2x} dx$.

5. (5 points each) Compute the limit of the sequence or show divergence:
(1) $\lim_{k\to\infty} \frac{e^k}{k}$. (2) $\lim_{n\to\infty} \frac{\sin(2n)}{n^2}$. (3) $\lim_{n\to\infty} \sum_{k=0}^{n} \frac{1}{e^k}$.

6. Given an infinite series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}},$$

show that the series is divergent using indicated methods: (1) (3 points) The comparison test. (You can use $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ is divergent without proof.) (2) (7 points) The integral test. (You should compute a divergent integral you need for comparison.)

7. (5 points) Show that the given series is convergent: $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$.

8. (10 points) Write down the degree 3 Taylor polynomial centered at 0:

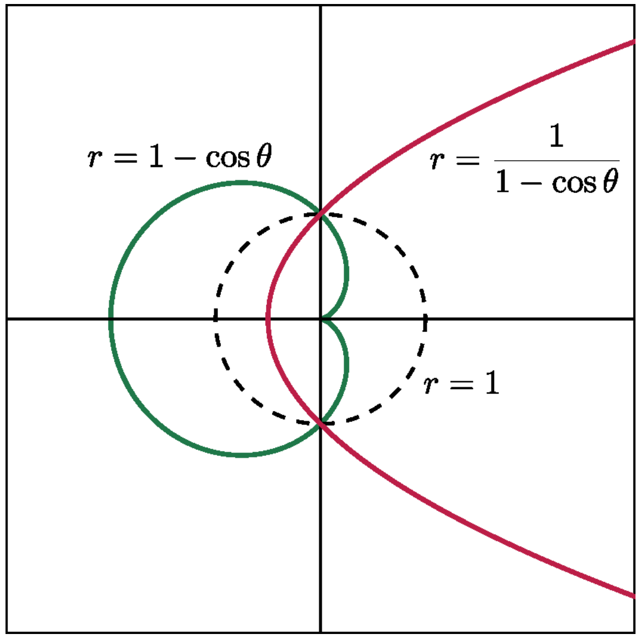
$$p_3(x) = \sum_{k=0}^{3} \frac{f^{(k)}(0)}{k!} x^k$$

for given $f(x) = 1 + \sin x$.

9. (10 points) Find the interval of convergence of the power series: $\sum_{n=2}^{\infty} \frac{2^n}{n!} x^n$. (Clearly mention whether your final answer is a(n) open, half-open, or closed interval!)

10. (1) (5 points) Let C be a circle of radius 3 centered at $(0, 3)$. Write the equation of C in the polar coodinate.

(2) (10 points) Calculate the enclosed area by the cardioid $r = 1 - \cos\theta$. (See overleaf for picture. It is the heart-shape closed curve.)


$r = 1 - \cos\theta$
$r = \frac{1}{1 - \cos\theta}$
$r = 1$