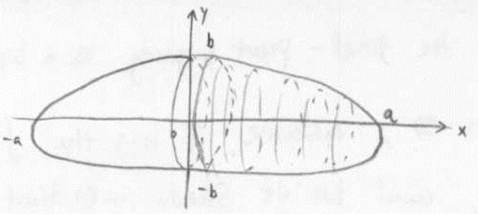
Solution to #2.

Assume $a \geq b$.



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \iff b^2x^2+a^2y^2=a^2b^2$$

Implicit differentiation: $2b^2x+2a^2yy'=0$

$$y'=-\frac{b^2x}{a^2y}$$

Surface area for ½ of ellipsoid of revolution:

$$A=\int_0^a 2\pi y\sqrt{1+(y')^2}\,dx \overset{(*)}{=} \int_0^a 2\pi y\sqrt{\left(\frac{1}{a^2y}\right)^2(a^4y^2+b^4x^2)}\,dx=\int_0^a 2\pi\,\frac{1}{a^2}\sqrt{a^4y^2+b^4x^2}\,dx=\Delta$$

$$(*)\quad 1+(y')^2=1+\frac{b^4x^2}{a^4y^2}=\left(\frac{1}{a^2y}\right)^2\left(a^4y^2+b^4x^2\right)$$

$$(**)\quad a^4y^2+b^4x^2=a^2(a^2b^2-b^2x^2)+b^4x^2=a^4b^2-(a^2b^2-b^4)x^2=(a^2b^2-b^4)\left(\frac{a^4b^2}{a^2b^2-b^4}-x^2\right)$$

Let $e^2=1-\dfrac{b^2}{a^2}$. $\quad a^2b^2e^2=a^2b^2-b^4$

$$\Delta=\int_0^a\frac{2\pi}{a^2}\cdot abe\sqrt{\frac{a^4b^2}{a^2b^2e^2}-x^2}\,dx=\int_0^{\sin^{-1}e}\frac{2\pi}{a}be\cdot\frac{a^2}{e^2}\cos^2\theta\,d\theta=\pi\frac{ab}{e}\int_0^{\sin^{-1}e}1+\cos 2\theta\,d\theta$$

$x=\dfrac{a}{e}\sin\theta$ $\quad \sin^{-1}e \,/\, 0$

$dx=\dfrac{a}{e}\cos\theta\,d\theta$

$e=\sin\theta$

$\cos\theta=\pm\sqrt{1-e^2}$

$$=\frac{ab}{e}\pi\left[\theta+\frac{1}{2}\sin 2\theta\Big|_0^{\sin^{-1}e}\right]$$

$$=\frac{ab}{e}\pi\left(\sin^{-1}e+e\cos\sin^{-1}e\right)$$

$$=\frac{ab\pi}{e}\left(\sin^{-1}e+\frac{eb}{a}\right)$$

Hence the area of ellipsoid of revolution (prolate):

$$\frac{2\pi ab}{e}\left(\sin^{-1}e+\frac{be}{a}\right).$$