

6. Given an infinite series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}},$$

show that the series is divergent using indicated methods: (1) (3 points) The comparison test. (You can use  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$  is divergent without proof.)

**Solution.** Notice that, for all positive integer  $n$ ,

$$\frac{1}{\sqrt{2n}} = \frac{1}{\sqrt{n+n}} \leq \frac{1}{\sqrt{n+1}}$$

Hence

$$\frac{1}{\sqrt{2}} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \leq \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$$

where the left-hand side is divergent by being a  $p$ -series with  $p = 1/2$ .