

Do not forget to save your work regularly both as yourname9.mws and yourname9b.mws every 10 minutes. Remember to include comments explaining what you are doing.

This lab has four problems. For each problem you should complete all the following steps:

- Define the function.
- Check if the function has a vertical asymptote by checking when the denominator is 0.
- Find the derivative of the function.
- Find the critical points.
- Find the second derivative of the function.
- Find the inflection points.
- List all the important x values, critical numbers, inflection numbers and vertical asymptotes in order and give a good domain of x values for the graph.
- If there is no asymptote, find the absolute maximum and absolute minimum of the function using the critical points you've found and the end points you chose in step f.
- Plot the function using the x values from step g and check that the graph does indeed have the correct critical points, inflection points, global maximum and minimum, and asymptotes. Write a comment or correct your work if necessary.
- Plot the graph again with a larger range of x and write comments.

By completing all of these steps you will have graphed many of the important qualities of the function.

EXAMPLE: Before doing the three problems complete this example here by hitting enter on each command line:

a)

```
> restart; f:= x -> x*(x-1)*(x+2)^2;
```

$$f := x \rightarrow x(x-1)(x+2)^2 \quad (1)$$

b)

There is no denominator so there are no vertical asymptotes.

c)

```
> D(f)(x);
```

$$(x-1)(x+2)^2 + x(x+2)^2 + 2x(x-1)(x+2) \quad (2)$$

```
> simplify(D(f)(x));
```

$$4x^3 + 9x^2 - 4 \quad (3)$$

d)

```
> solve(D(f)(x)=0);
```

$$-2, -\frac{1}{8} + \frac{1}{8}\sqrt{33}, -\frac{1}{8} - \frac{1}{8}\sqrt{33} \quad (4)$$

These are the three critical points and I will use evalf to see their decimal approximations:

```
> evalf(solve(D(f)(x)=0));
```

$$-2., 0.5930703309, -0.8430703309 \quad (5)$$

e)

```
> D(D(f));
```

$$x \rightarrow 2(x+2)^2 + 4(x-1)(x+2) + 4x(x+2) + 2x(x-1) \quad (6)$$

f)

```
> solve(D(D(f))(x)=0);
```

$$0, -\frac{3}{2} \quad (7)$$

There are inflection points at 0 and -1.5.

g)

The important x values in order are approximately:

-2, -1.5, -.84, 0, .59

We can choose the domain to be x in [-3,1].

h)

To find the absolute max and min we check the endpoints:

```
> f(-3);f(1);
```

$$\begin{array}{l} 12 \\ 0 \end{array} \quad (8)$$

and the critical points:

```
> f(-2);f(-.84); f(.59);
```

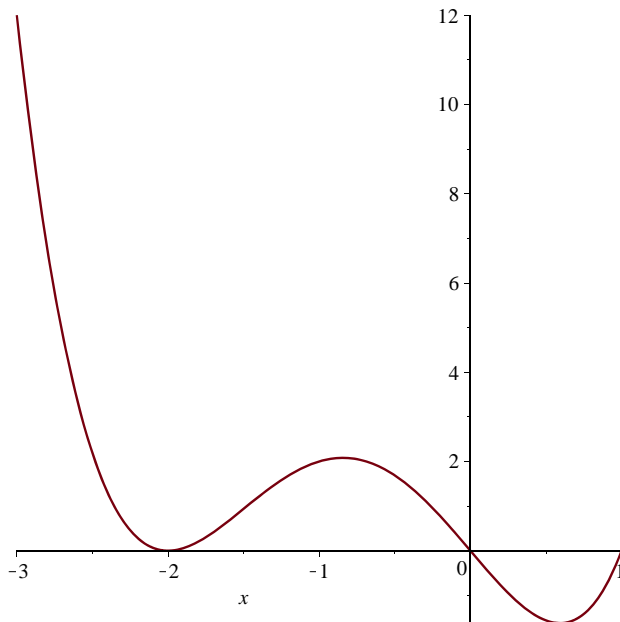
$$\begin{array}{l} 0 \\ 2.07975936 \\ -1.62268939 \end{array} \quad (9)$$

So the absolute max occurs at x=-3 and has a y value of 12.

The absolute minimum occurs near x=.59 and has a y value of -1.6.

i)

```
> plot(f(x), x=-3..1);
```



You fill in the comments for this part.

Do the following problems on yourname11.mws. Be sure to put the letters marking each step and to add comments just as in the example. Cutting and pasting commands will help you work faster.

Problem 1: Graph $f(x) = (x-50)(x-55)(x-60)^2$

```
> f:=x->(x-50)*(x-55)*(x-60)^2;
      f:=x→(x−50)(x−55)(x−60)2 (10)
```

```
> No vertical asymptotes.
```

```
> D(f);
      x→(x−55)(x−60)2+ (x−50)(x−60)2+ 2(x−50)(x−55)(x−60) (11)
```

```
> crit:=solve(D(f)(x)=0,x);
      crit:=60, 435/8 + 5/8√17, 435/8 - 5/8√17 (12)
```

```
> D(D(f));
      x→2(x−60)2+ 4(x−55)(x−60)+ 4(x−50)(x−60)+ 2(x−50)(x−55) (13)
```

```
> infl:=solve(D(D(f))(x)=0,x);
```

$$\text{infl} := \frac{225}{4} + \frac{5}{12} \sqrt{33}, \frac{225}{4} - \frac{5}{12} \sqrt{33} \quad (14)$$

```
> imppts:=[crit[1],crit[2],crit[3],infl[1],infl[2],50];
  evalf_imppts:=evalf([crit[1],crit[2],crit[3],infl[1],infl[2]]);
```

$$\text{imppts} := \left[60, \frac{435}{8} + \frac{5}{8} \sqrt{17}, \frac{435}{8} - \frac{5}{8} \sqrt{17}, \frac{225}{4} + \frac{5}{12} \sqrt{33}, \frac{225}{4} - \frac{5}{12} \sqrt{33}, 50 \right]$$

$$\text{evalf_imppts} := [60., 56.95194102, 51.79805898, 58.64356777, 53.85643223] \quad (15)$$

```
> valimppts:=map(f,imppts);evalf(map(f,imppts));
```

$$\text{valimppts} := \left[0, \left(\frac{35}{8} + \frac{5}{8} \sqrt{17} \right) \left(-\frac{5}{8} + \frac{5}{8} \sqrt{17} \right) \left(-\frac{45}{8} + \frac{5}{8} \sqrt{17} \right)^2, \left(\frac{35}{8} - \frac{5}{8} \sqrt{17} \right) \left(-\frac{5}{8} - \frac{5}{8} \sqrt{17} \right) \left(-\frac{45}{8} - \frac{5}{8} \sqrt{17} \right)^2, \left(\frac{25}{4} + \frac{5}{12} \sqrt{33} \right) \left(\frac{5}{4} + \frac{5}{12} \sqrt{33} \right) \left(-\frac{15}{4} + \frac{5}{12} \sqrt{33} \right)^2, \left(\frac{25}{4} - \frac{5}{12} \sqrt{33} \right) \left(\frac{5}{4} - \frac{5}{12} \sqrt{33} \right) \left(-\frac{15}{4} - \frac{5}{12} \sqrt{33} \right)^2, 0 \right]$$

$$[0., 126.0722496, -387.3027184, 57.94501693, -166.4519615, 0.] \quad (16)$$

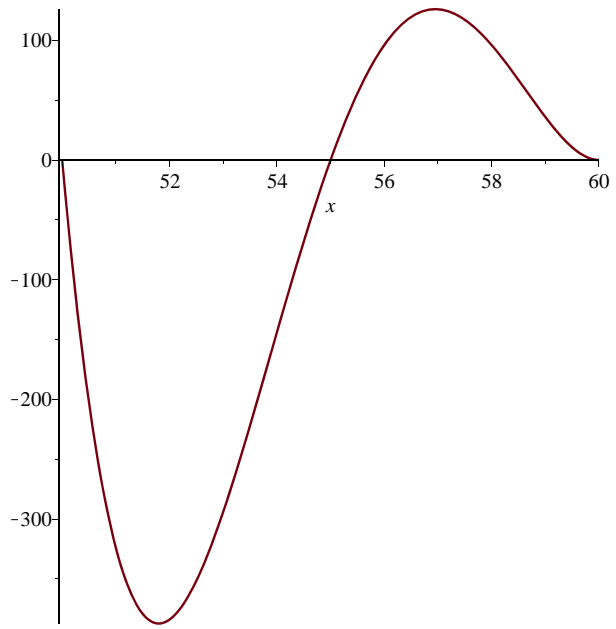
```
> AbsMax:=(imppts[2],valimppts[2]);
```

$$\text{AbsMax} := \frac{435}{8} + \frac{5}{8} \sqrt{17}, \left(\frac{35}{8} + \frac{5}{8} \sqrt{17} \right) \left(-\frac{5}{8} + \frac{5}{8} \sqrt{17} \right) \left(-\frac{45}{8} + \frac{5}{8} \sqrt{17} \right)^2 \quad (17)$$

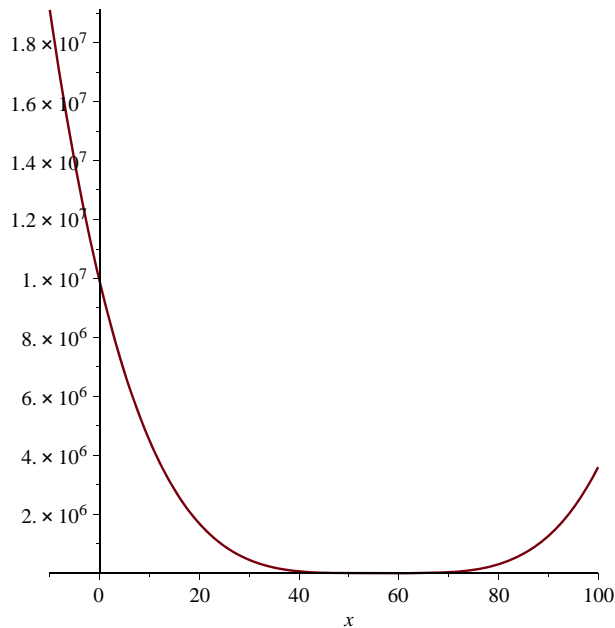
```
> AbsMin:=(imppts[3],valimppts[3]);
```

$$\text{AbsMin} := \frac{435}{8} - \frac{5}{8} \sqrt{17}, \left(\frac{35}{8} - \frac{5}{8} \sqrt{17} \right) \left(-\frac{5}{8} - \frac{5}{8} \sqrt{17} \right) \left(-\frac{45}{8} - \frac{5}{8} \sqrt{17} \right)^2 \quad (18)$$

```
> plot(f(x),x=50..60);
```



```
> plot(f(x),x=-10..100);
```



Problem 2: Graph $f(x)=(x-2)^2(x-5)^2(x+1)^2$

> f:=x->(x-2)^2*(x-5)^2*(x+1)^2;

$$f := x \rightarrow (x-2)^2 (x-5)^2 (x+1)^2 \quad (19)$$

> No vertical asymptotes.

> D(f);

$$x \rightarrow 2(x-2)(x-5)^2(x+1)^2 + 2(x-2)^2(x-5)(x+1)^2 + 2(x-2)^2(x-5)^2(x+1) \quad (20)$$

> crit:=solve(D(f)(x)=0,x);

$$crit := -1, 2, 5, 2 + \sqrt{3}, 2 - \sqrt{3} \quad (21)$$

> D(D(f));

$$x \rightarrow 2(x-5)^2(x+1)^2 + 8(x-2)(x-5)(x+1)^2 + 8(x-2)(x-5)^2(x+1) + 2(x-2)^2(x+1)^2 + 8(x-2)^2(x-5)(x+1) + 2(x-2)^2(x-5)^2 \quad (22)$$

> infl:=solve(D(D(f))(x)=0,x);

$$infl := 2 - \frac{1}{5} \sqrt{90 - 15\sqrt{21}}, 2 + \frac{1}{5} \sqrt{90 - 15\sqrt{21}}, 2 - \frac{1}{5} \sqrt{90 + 15\sqrt{21}}, 2 \quad (23)$$

$$+ \frac{1}{5} \sqrt{90 + 15\sqrt{21}}$$

```
> evalf(crit);evalf(infl);
```

$$\begin{aligned} & -1., 2., 5., 3.732050808, 0.267949192 \\ & 1.077799055, 2.922200945, -0.519830434, 4.519830434 \end{aligned} \quad (24)$$

```
> impval:=[crit[1],crit[2],crit[3],crit[4],crit[5],infl[1],infl[2],infl[3],infl[4]];
```

$$\text{impval} := \left[-1, 2, 5, 2 + \sqrt{3}, 2 - \sqrt{3}, 2 - \frac{1}{5} \sqrt{90 - 15\sqrt{21}}, 2 + \frac{1}{5} \sqrt{90 - 15\sqrt{21}}, 2 - \frac{1}{5} \sqrt{90 + 15\sqrt{21}}, 2 + \frac{1}{5} \sqrt{90 + 15\sqrt{21}} \right] \quad (25)$$

```
> valimpval:=map(f,impval);
```

$$\begin{aligned} \text{valimpval} := & \left[0, 0, 0, 3(-3 + \sqrt{3})^2(3 + \sqrt{3})^2, 3(-3 - \sqrt{3})^2(3 - \sqrt{3})^2, \frac{1}{25}(90 - 15\sqrt{21}) \left(-3 - \frac{1}{5}\sqrt{90 - 15\sqrt{21}}\right)^2 \left(3 - \frac{1}{5}\sqrt{90 - 15\sqrt{21}}\right)^2, \frac{1}{25}(90 - 15\sqrt{21}) \left(-3 + \frac{1}{5}\sqrt{90 - 15\sqrt{21}}\right)^2 \left(3 + \frac{1}{5}\sqrt{90 - 15\sqrt{21}}\right)^2, \frac{1}{25}(90 + 15\sqrt{21}) \left(-3 - \frac{1}{5}\sqrt{90 + 15\sqrt{21}}\right)^2 \left(3 - \frac{1}{5}\sqrt{90 + 15\sqrt{21}}\right)^2, \frac{1}{25}(90 + 15\sqrt{21}) \left(-3 + \frac{1}{5}\sqrt{90 + 15\sqrt{21}}\right)^2 \left(3 + \frac{1}{5}\sqrt{90 + 15\sqrt{21}}\right)^2 \right] \end{aligned} \quad (26)$$

```
> evalf(valimpval);
```

$$[0., 0., 0., 107.9999999, 107.9999999, 56.48301812, 56.48301812, 44.60498192, 44.60498192] \quad (27)$$

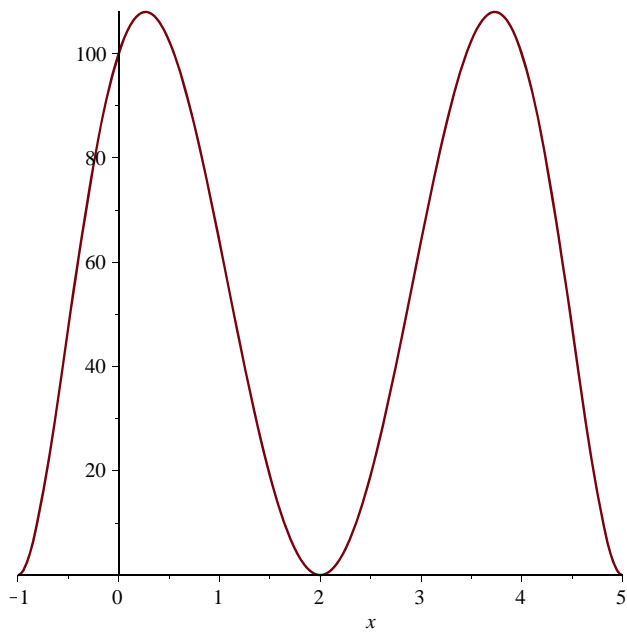
```
> AbsMax:=(impval[4],valimpval[4]);
```

$$\text{AbsMax} := 2 + \sqrt{3}, 3(-3 + \sqrt{3})^2(3 + \sqrt{3})^2 \quad (28)$$

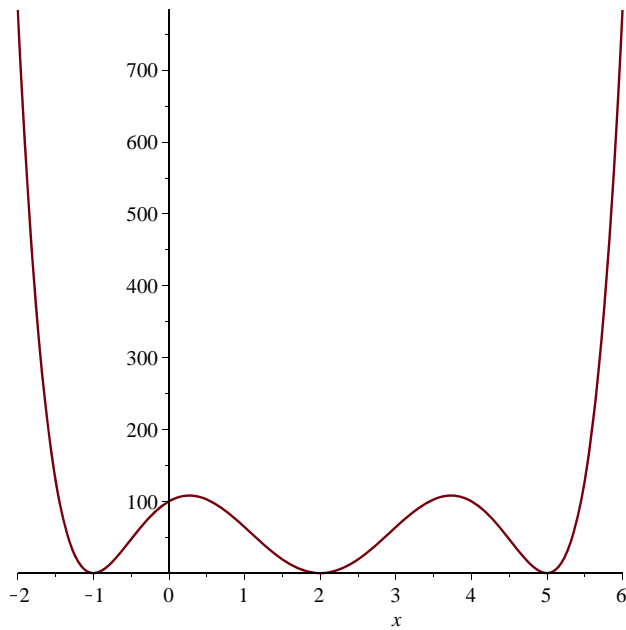
```
> AbsMin:=(impval[1],valimpval[1]);
```

$$\text{AbsMin} := -1, 0 \quad (29)$$

```
> plot(f(x),x=-1..5);
```



```
> plot(f(x),x=-2..6);
```

Problem 3: Graph $f(x) = (x^2 + 1)^{1/100}$

> `f:=x->(x^2+1)^(1/100);`

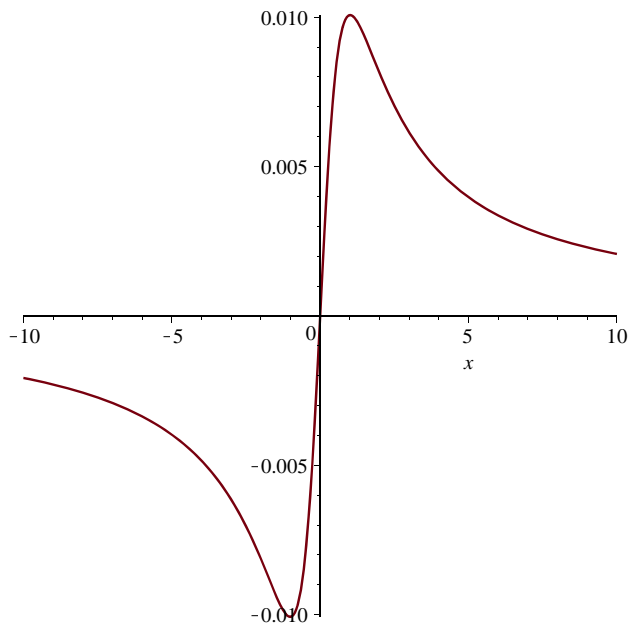
$$f := x \rightarrow (x^2 + 1)^{\frac{1}{100}}$$

> **No vertical asymptotes.**

> `D(f); plot(D(f)(x),x=-10..10);`

$$x \rightarrow \frac{1}{50} \frac{x}{(x^2 + 1)^{\frac{99}{100}}}$$

(30)



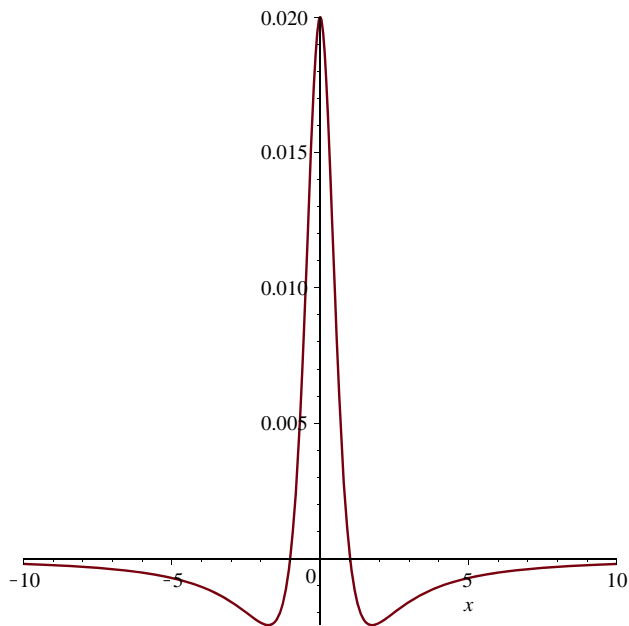
```
> crit:=solve(D(f)(x)=0,x);
```

```
crit := 0
```

```
> D(D(f)); plot(D(D(f))(x),x=-10..10);
```

$$x \rightarrow -\frac{99}{2500} \frac{x^2}{(x^2+1)^{\frac{199}{100}}} + \frac{1}{50} \frac{1}{(x^2+1)^{\frac{99}{100}}}$$

(31)



```
> infl:=[fsolve(D(D(f))(x)=0,x=-5..0),fsolve(D(D(f))(x)=0,x=0..5)];
      infl := [-1.010152545, 1.010152545] (32)
```

```
> evalf(crit);evalf(infl);
      0.
      [-1.010152545, 1.010152545] (33)
```

```
> impval:=[-2,crit[1],infl[1],infl[2],2];
      impval := [-2, 0, -1.010152545, 1.010152545, 2] (34)
```

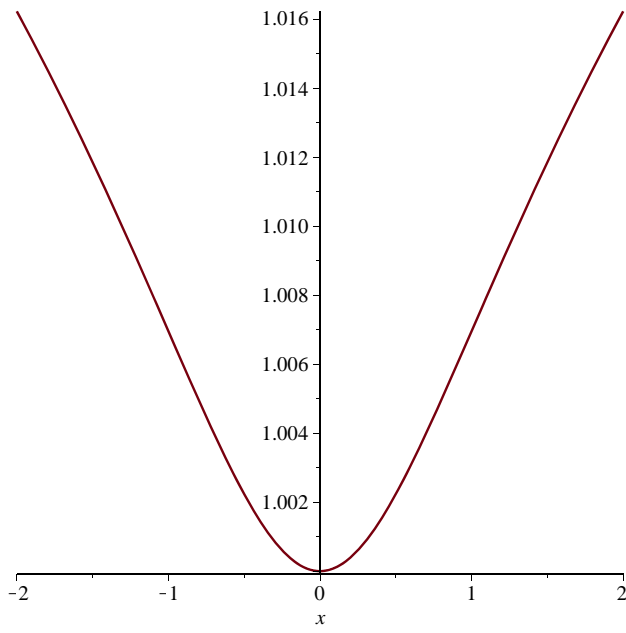
```
> valimpval:=map(f,impval);
      valimpval := [51/100, 1, 1.007057785, 1.007057785, 51/100] (35)
```

```
> evalf(valimpval);
      [1.016224591, 1., 1.007057785, 1.007057785, 1.016224591] (36)
```

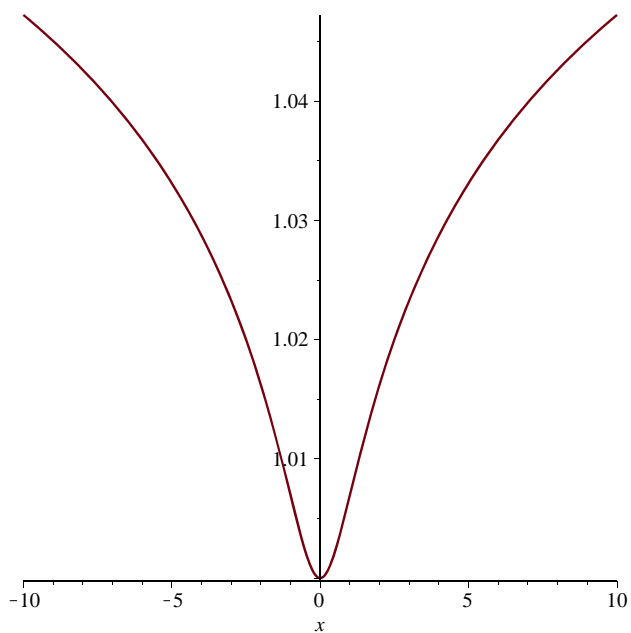
```
> AbsMax:=(impval[1],valimpval[1]);
      AbsMax := -2, 51/100 (37)
```

```
> AbsMin:=(impval[2],valimpval[2]);
      AbsMin := 0, 1 (38)
```

```
> plot(f(x),x=-2..2);
```



```
> plot(f(x),x=-10..10);
```



Problem 4: Graph $f(x) = (4 + 4x + x^2)/x$

> $f := x \rightarrow (4 + 4 * x + x^2) / x;$

$$f := x \rightarrow \frac{4 + 4x + x^2}{x}$$

(39)

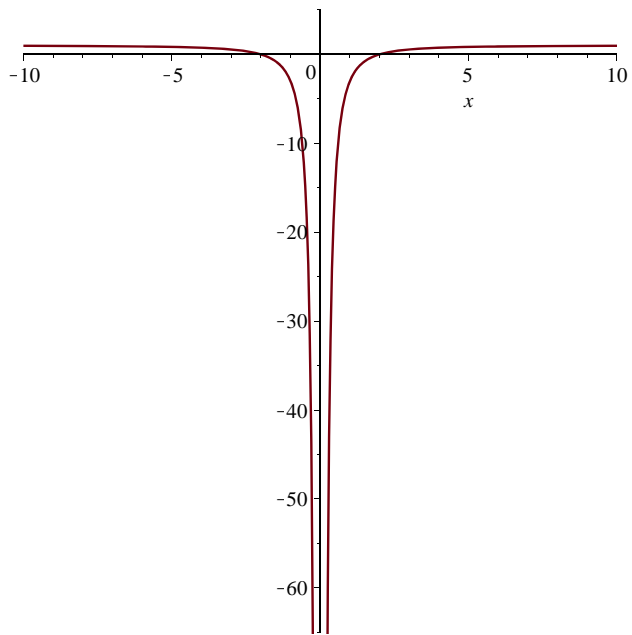
> Vertical asymptote $x=0$.

> $D(f);$

$$x \rightarrow \frac{2x + 4}{x} - \frac{4 + 4x + x^2}{x^2}$$

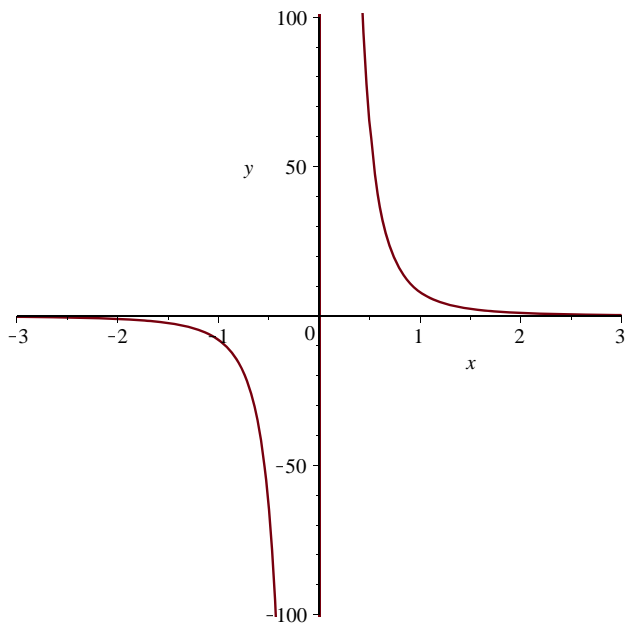
(40)

> $crit := solve(D(f)(x) = 0, x); plot(D(f)(x));$
 $crit := 2, -2$



```
> D(D(f)); plot(D(D(f))(x),x=-3..3, y=-100..100);
```

$$x \rightarrow \frac{2}{x} - \frac{2(2x+4)}{x^2} + \frac{2(4+4x+x^2)}{x^3}$$



```
> infl:=solve(D(D(f))(x)=0,x);
                                infl := (41)
```

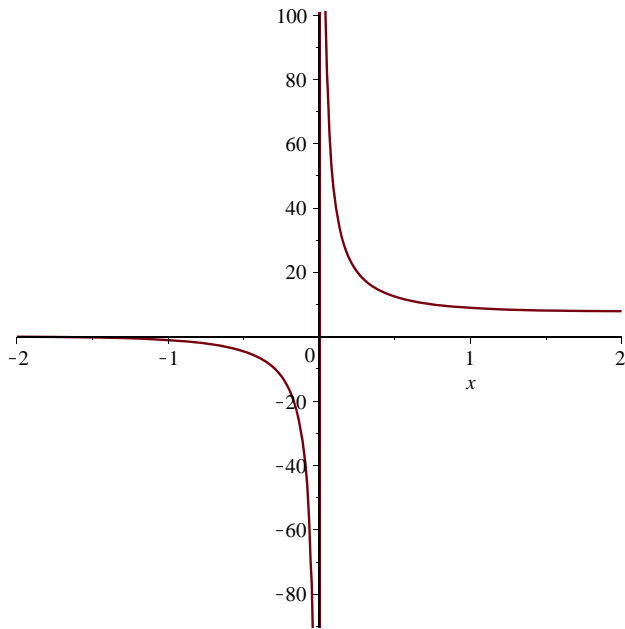
```
> impval:=[crit[2],crit[1]];
                                impval := [-2, 2] (42)
```

```
> valimpval:=map(f,impval);
                                valimpval := [0, 4] (43)
```

```
> RelMax:=(impval[2],valimpval[2]);
                                RelMax := 2, 4 (44)
```

```
> RelMin:=(impval[1],valimpval[1]);
                                RelMin := -2, 0 (45)
```

```
> plot(f(x),x=-2..2);
```



Exploration: Try to graph $f(x) = (4 + 4x + x^2) / x^2$. It will not work nicely. Focus on the inflection points and critical points and discuss why there is trouble graphing this function. You may prefer to graph $f(x) = \tan(x)$ and discuss the difficulties with this graph.

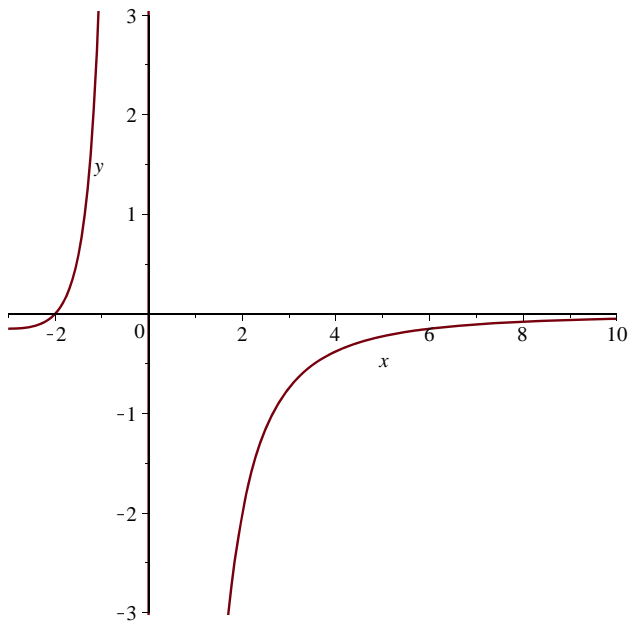
> $f := x \rightarrow (4 + 4 * x + x^2) / x^2$;

$$f := x \rightarrow \frac{4 + 4x + x^2}{x^2}$$

(46)

> $D(f)$; $\text{plot}(D(f)(x), x = -3..10, y = -3..3)$;

$$x \rightarrow \frac{2x + 4}{x^2} - \frac{2(4 + 4x + x^2)}{x^3}$$



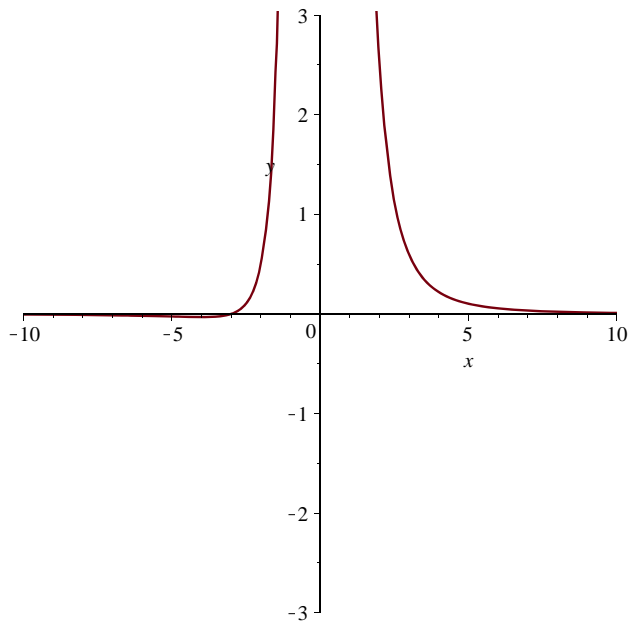
```
> crit:=solve(D(f)(x)=0,x);
```

```
crit := -2
```

(47)

```
> D(D(f));plot(D(D(f))(x),x=-10..10, y=-3..3);
```

$$x \rightarrow \frac{2}{x^2} - \frac{4(2x+4)}{x^3} + \frac{6(4+4x+x^2)}{x^4}$$



```
> infl:=solve(D(D(f))(x)=0,x);
                                infl := -3
```

```
> impval:=[infl,crit];
                                impval := [-3, -2]
```

```
> valimpval:=map(f,impval);
                                valimpval := [1/9, 0]
```

```
> RelMax:= none Absolute Max:=infinity
> AbsMin:=(impval[2],valimpval[2]);
                                AbsMin := -2, 0
```

```
> plot(f(x),x=-3..0,y=-3..10);
```

