MAT 155 Project II Spring 2014
Functions and Graphing

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If you missed the first project, you must complete it in order to be able to do this one. Ask your teacher if you should do it now or if you should sit next to someone who was here last week.(Do the latter!) In the future, if you miss a lab, you must make up the work before coming to class.

Before starting this project you should have reviewed P1-P3 for homework. If you have not read these sections, please refer to them as necessary. In future projects it will be assumed that you have already done the reading before coming to class.

Saving this file:
Before you begin working you should save this file as **yourname2.mws** by selecting "File" on the far left of the top bar of Maple V and choosing "**Save As**". A window will appear and you will fill in the file name. You should regularly save this file using the "Save As". You should also save the file with a second name **yourname2b.mws** using the "Save As" **so that you have a backup** in case your file is damaged. The computer will tell you that the file already exists and you will click on "Yes" that you agree to overwrite it. If you just use the "Save" instead of the "Save As" then the file will be saved with whichever name is on the top bar of this window.

Put your name **on the top of this file** by clicking your mouse right below the title and then typing your name. Then return here using the down arrow.

Problem 1: Defining functions:

A function is a mapping from a domain to a range, it assigns one value for every input x.
For example the square root function maps x to the square root of x. We can also define a function which maps x to (x+3)/5. To do so you must type:

```
> f:= x -> (x+3)/5;
```

$$f := x \rightarrow \frac{1}{5}\,x + \frac{3}{5} \qquad \textbf{(1)}$$

Hit enter on the line so that Maple learns the definition. Now you can find the value of f at a few different values of x. For example:

```
> f(2);
```

$$1 \qquad \textbf{(2)}$$

```
> f(7);
```

$$2 \qquad \textbf{(3)}$$

```
> f(1);
```

$$\frac{4}{5} \qquad \textbf{(4)}$$

To get a decimal expansion from a fraction, use the evalf command:

```
> evalf(f(1));
```

$$0.8000000000 \qquad \textbf{(5)}$$

You could also find f at Pi=3.14159....

```
> f(Pi);
```

$$\frac{1}{5}\pi + \frac{3}{5} \quad \textbf{(6)}$$

```
> evalf(f(Pi));
```

$$1.228318531 \quad \textbf{(7)}$$

Now define a function g(x)=(x+30)(x-20). Don't forget to use a star (above the 8) for the multiplication.

```
> g:=x -> (x+30)*(x-20);
```

$$g := x \rightarrow (x+30)\,(x-20) \quad \textbf{(8)}$$

Evaluate the function at 20 and at -30. Try some other x's as well.

```
> g(20);
```

$$0 \quad \textbf{(9)}$$

```
> g(-30);
```

$$0 \quad \textbf{(10)}$$

```
> g(0);
```

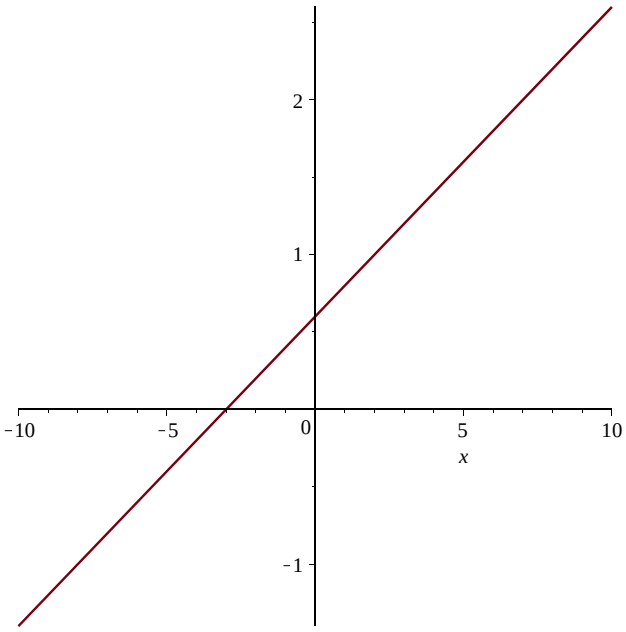
$$-600 \quad \textbf{(11)}$$

It is time to save your work. Got to the top using the arrow key if you need to review how to save the file. Remember you must save it twice, once as yourname2.mws and once as yourname2b.mws.

Problem 2: Graphing Functions:

To graph a function on a computer you must tell it what x values you are interested in viewing. Here is the command for graphing a function f from x= -10 to 10.

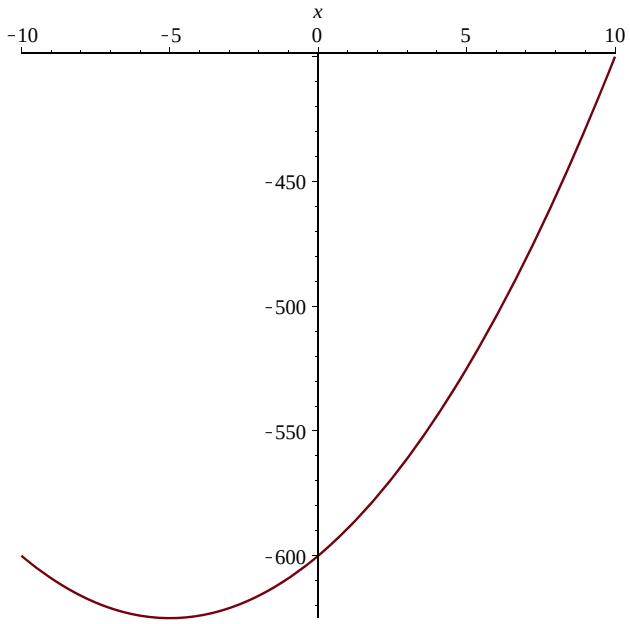
```
> plot(f(x), x=-10..10);
```



Notice that the y axis is scaled differently than the x axis.

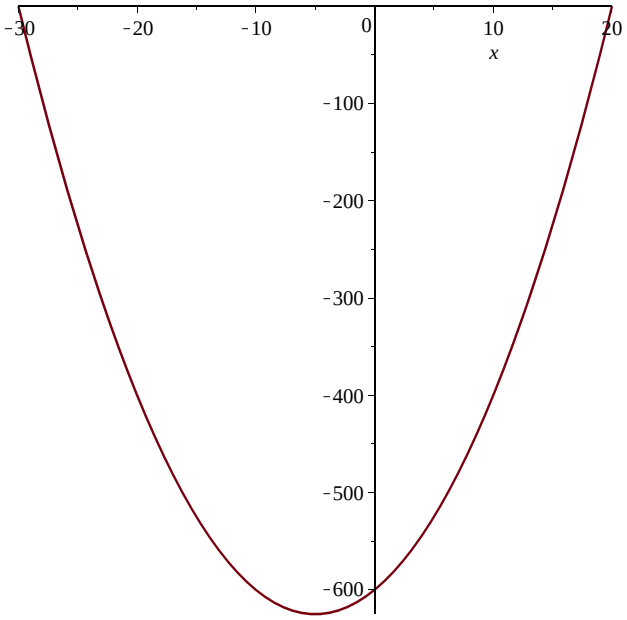
Now try to graph the function g from x=-10 to 10.

```
> plot(g(x), x=-10..10);
```



You'll notice that g is never 0 in this graph. But in Problem 1 you saw that it was 0 at 20 and at -30. So perhaps a graph that includes x=20 and x=-30 would be more interesting. Plot g again using a different domain of x values so that you see where it crosses the x axis. It should cross twice.

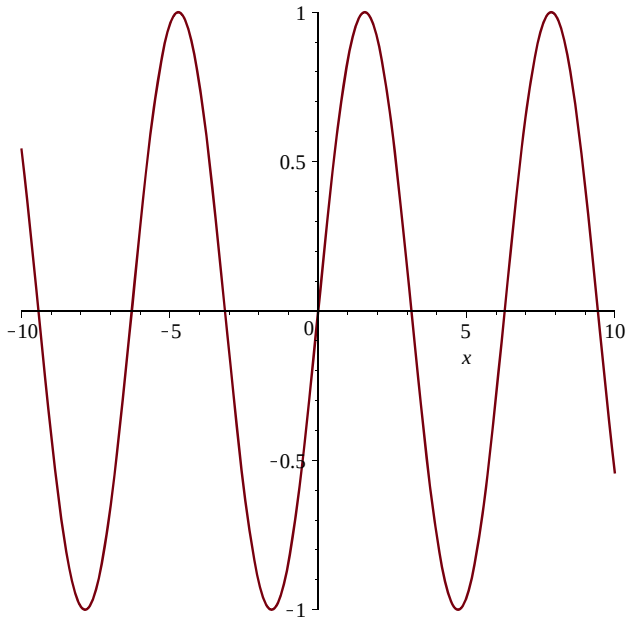
```
> plot(g(x), x=20..-30);
```



Notice that the order of endpoints in the plot command doesn't matter.

Now graph sin(x) from -10 to 10. Where does sin(x) cross 0?

```
> plot(sin(x), x=-10..10);
```



It should cross the x-axis at each multiple of Pi.

Find the value of sin(x) at x=Pi. Remember that in calculus we use radians not degrees and that Pi radians is 180 degrees. Review radians for homework (A.3 in the text)

```
> sin(Pi);
```

0 **(12)**

Now find an x such that sin(x) =1 by locating one of the high points on the graph. *Use the **trace feature** to do this by taking the mouse and clicking on the high point with the arrow tip. A pair of numbers will appear on the upper left corner of your screen. The first is the x value of the point you clicked on and the second is the y value. The second should be the number one. Write down the first. Then verify that the sine of that number is one.*

```
> sin(1.5624022);
```

0.9999647695 **(13)**

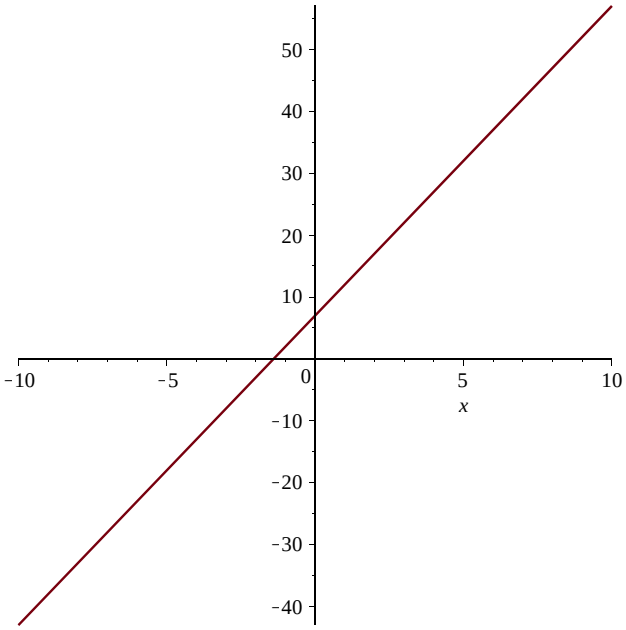
If you don't see the coordinate right next to your mouse cursor, then you should choose right mode in "plot."

It is time to save your work. Remember you must save it twice.

Problem 3: Graphing Lines

Define a function f(x)=5x+7 and graph it. Don't forget to use the multiplication symbol.

```
> plot(5*x+7, x=-10..10);
```



Recall that the slope of the line can be found by identifying two points on the line (a,b) and (c,d). *Use the **trace feature** to do this by taking the mouse and clicking on the line with the arrow tip. A pair of numbers will appear on the upper left corner of your screen. Write them down. Then click on another point in the line and copy down.* The first pair can be (a,b) and the second can be (c,d). Compute the slope of the line by computing the change in y coordinates over change in x coordinates=(d-b)/(c-a). You should get 5.

```
> (a,b):=(1,12);
```

$$a, b := 1, 12 \qquad (14)$$

```
> (c, d) := (2, 17);
```

$$c, d := 2, 17 \qquad (15)$$

```
> (d-b)/(c-a);
```

$$5 \qquad (16)$$

Notice that this is different from the following:

```
> (a, b) = (1, 12);
> (c, d) = (2, 17);
> (d-b)/(c-a);
```

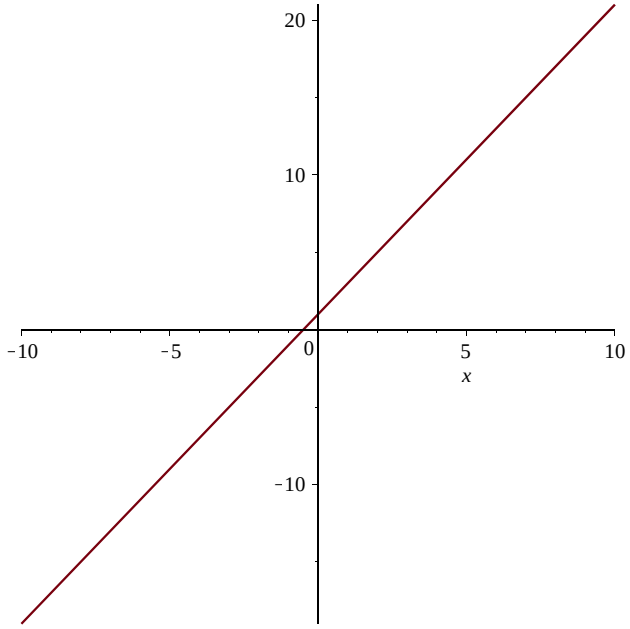
To graph a line with a given slope, m, and a point (a,b), you can find a line through that point using the formula **y - b = m (x -a)** However, this formula is an equation not a function. A function should always have y by itself on one side so that when you input an x value you get a y value. So you rewrite this as **y = m(x-a) + b** and fill in the numbers for the point (a,b) and the slope m. For example, the function whose graph is a line through (1,3) with a slope 2 is:

```
> f:= x -> 2*(x-1) +3;
```

$$f := x \rightarrow 2x + 1 \qquad (17)$$

Graph this function for x in [-10,10].

```
> plot(f(x),x=-10..10);
```



Define a function whose graph is a line which goes through (5,30) with slope -3. **Hint: Point**

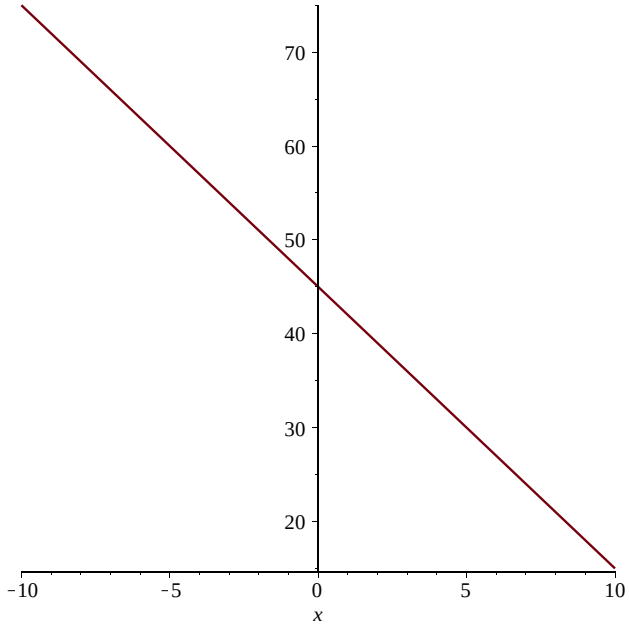
Slope Formula

```
> f:=x->-3*(x-5)+30;
```

$$f := x \rightarrow -3x + 45 \qquad \textbf{(18)}$$

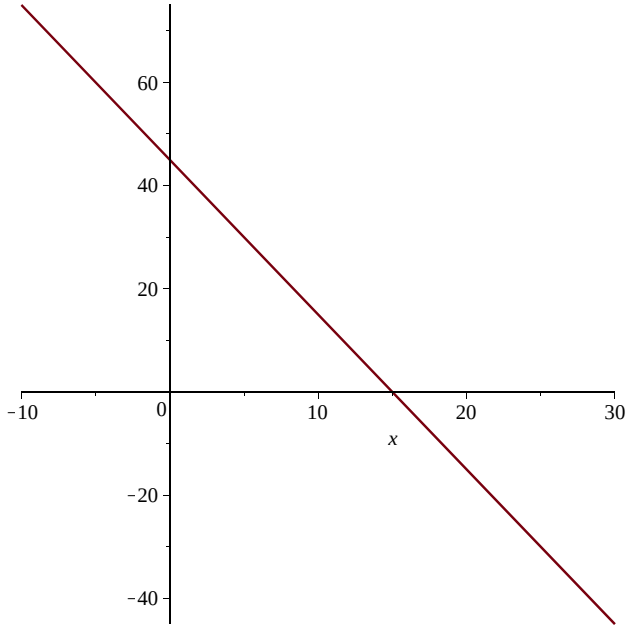
Graph the function for x in [-10,10].

```
> plot(f(x),x=-10..10);
```



Can you guess where the graph crosses the x axis, and graph it again below changing your x domain to include that point. ***You do not have to copy over the entire plot command by hand.*** *Hold the left mouse button down as you trace the part of the command you want to copy so it should be highlighted. Then go to the "Edit" on the top bar and select "copy". Then click below on [>, go back to "Edit" and select "Paste" and it will appear. Then you can just change the x part.*

```
> plot(f(x),x=-10..30);
```

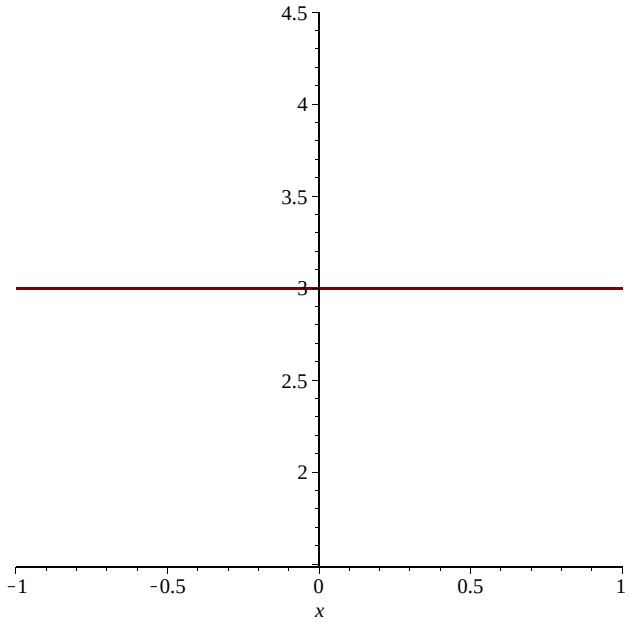


If the slope is positive then the function is increasing (and the graph goes upwards from left to right) and if the slope is negative then the function is decreasing (and the graph does downward from left to right). Define a function whose graph is just a horizontal line. If necessary try a few times.

```
> f:=x-> 3;
```

$$f := x \rightarrow 3 \qquad \textbf{(19)}$$

```
> plot(f(x),x=-1..1);
```



It is time to save your work. Remember you must save it twice.

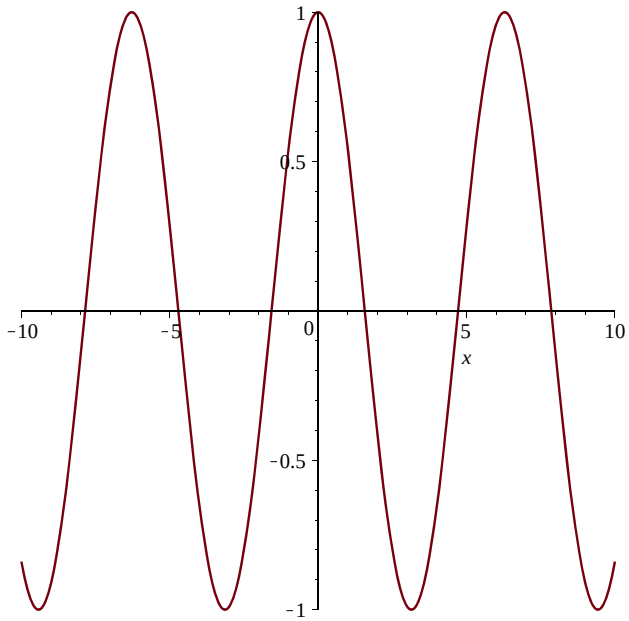
Problem 4: Odd and Even Functions:

Define f(x)=cos(x) and graph it with x in [-10,10].

```
> f:=x->cos(x);
```

$$f := x \rightarrow \cos(x)$$ **(20)**

```
> plot(f(x),x=-10..10);
```

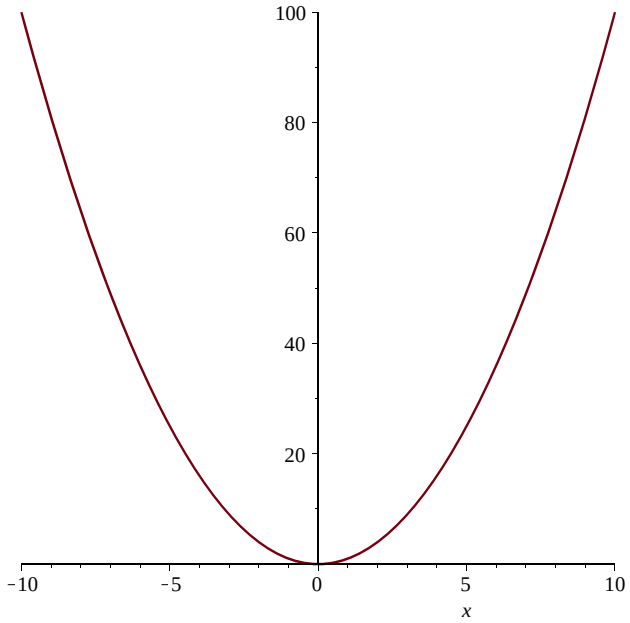


Define g(x)=x^2 and graph it with x in [-10,10].

```
> g:=x->x^2;
```

$$g := x \rightarrow x^2 \qquad \textbf{(21)}$$

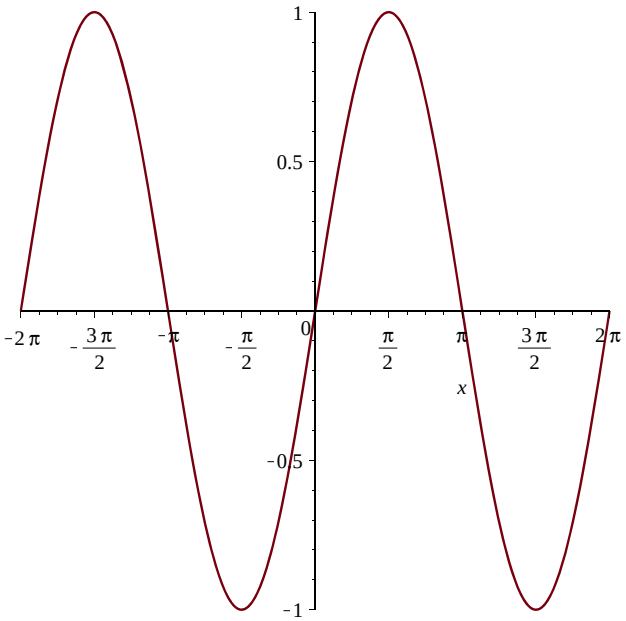
```
> plot(g(x),x=-10..10);
```



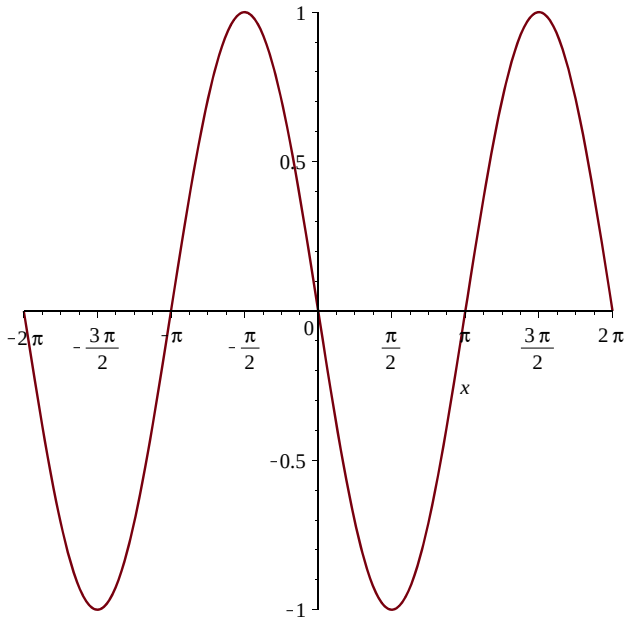
Notice that both of these graphs can be reflected symmetrically across the y axis (like a mirror). Thus they are called **even functions**. That is f(x)=f(-x). Graph f(-x) and g(-x) to see that they are the same as the graphs of f(x) and f(-x).

Now graph sin(x) and sin(-x). *Again use the copy and the paste to avoid typing the commands twice.*

```
> plot(sin(x),x=-2*Pi..2*Pi);
```




```
> plot(sin(-x),x=-2*Pi..2*Pi);
```



Is sin(x) an even function? No, these graphs are not the same. In fact sin(-x) = - sin(x) ! So sin(x) is an **odd function.**

It is time to save your work. Remember you must save it twice.

Exploration: Every project ends with exploration. Before starting the exploration you should make sure that you have completed all of the problems. Once you have completed them you can make up similar problems or do recommended exploration problems. If you miss a lab, you must catch up on the previous labs problems to learn all the commands and ideas that you missed. The exploration need not have been completed to continue with the next project. Your professor may, however, require you to complete the exploration for grading purposes.

Recommended exploration problems:

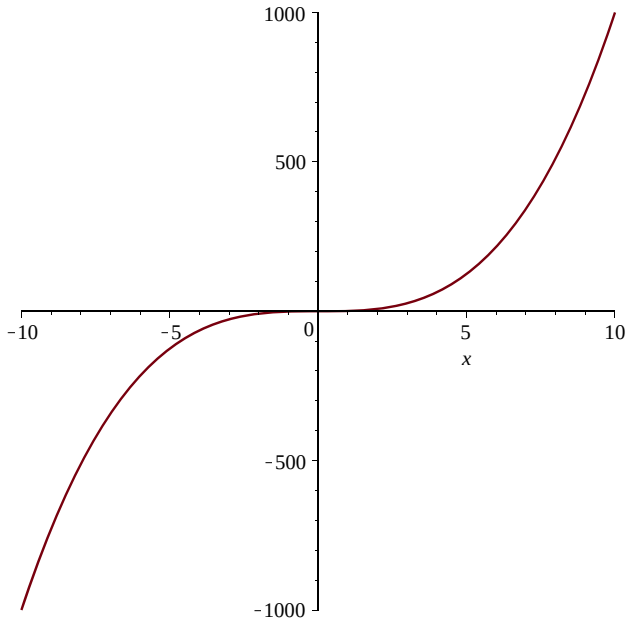
A) Shifts: Let f(x)=x^3. Then graph f(x), f(x+2), and f(x)+2 and compare them. Now try to shift the

graph of the function other ways. For example shift it to the right by 3, shift it to the left by 4, shift it up by 5, and shift it down by 6. Now change f to another function and hit enter on all your lines to see how the graphs change. Just hit enter below to get more prompts [>.

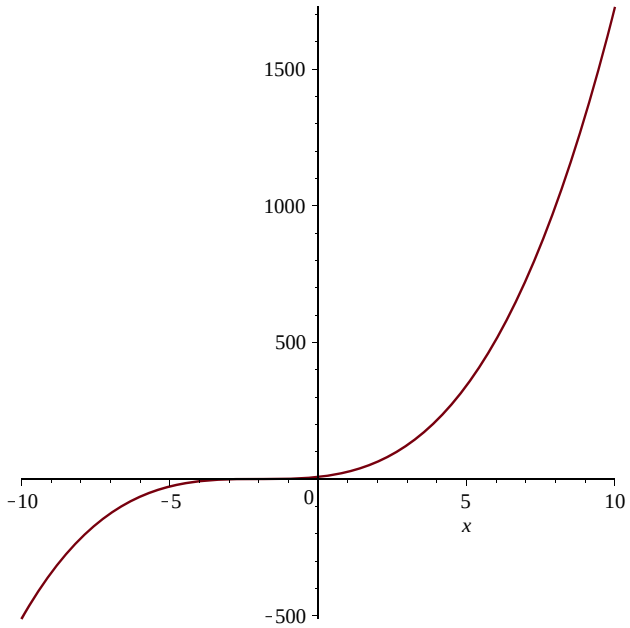
```
> f := x → x^3;
```

$$f := x \to x^3 \qquad \textbf{(22)}$$

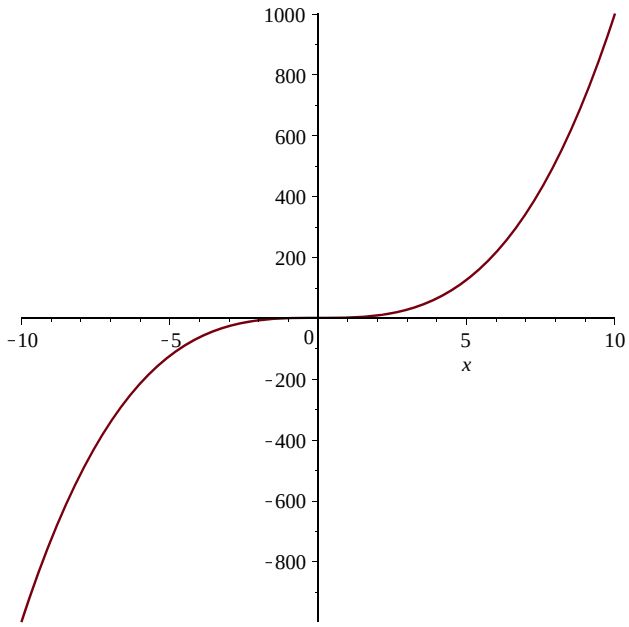
```
> plot( f(x), x = -10..10);
```




```
> plot( f(x + 2), x = -10..10);
```



> $plot(f(x)+2, x=-10..10);$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