MAT 155 Project III Spring 2014

Examining Graphs

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Do not work on this file. This is just the list of problems. **Open your own file** by selecting "File" on the Maple V bar and choosing "New". Save the new file as **yourname3.mws** and as **yourname3b.mws** regularly using the "save as" command. **Be careful not to save this file with that name or you will overwrite your work!** This window should always have 155.00.03.mws printed on the top. If you click on the button on this window next to the X (the one marked with two boxes) then you will be able to see this file and your file at the same time. You can adjust the sizes of the windows displaying each file as needed.

Do not hit return on this file or the computer will remember it when working on your file and may overwrite variables and functions that you've defined.

Sign your name as a comment at the top of your file by backspacing in front of the prompt and typing it in. Also write Project III and the names of any other students who are working with you.

At the start of each problem in your file, backspace in front of the prompt and type the Problem Number 1 or Problem Number 2 etc. Then hit return and on the first line with a prompt type

```
> restart;
```

This will clear all previous work. *Also remember to save your file with both names.*

Do all the problems in order. Ask a student or the teacher if you are having trouble.

Problem 1: Solving a quadratic equation.

Define a function f(x)= x^2+5x+6 using the command:

```
> f:= x -> x^2 +5*x+6;
```

$$f := x \rightarrow x^2 + 5\,x + 6 \quad (1)$$

To find out where f(x)=0, there are several methods. You can factor it:

```
> factor(f(x));
```

$$(x+3)\,(x+2) \quad (2)$$

So that you see that it is zero at the two roots: x = 3 and x = 2. Note that if f(x) has no rational solutions then Maple cannot factor it. Try g(x)=x^2-2 and h(x)= x^2+4x+6.

You could also use the command

```
> solve(f(x)=0, x);
```

$$-2, -3 \quad (3)$$

> $g := x \rightarrow x^2 - 2;$

$$g := x \rightarrow x^2 - 2 \quad (4)$$

> $h := x \rightarrow$ x^2 + 4 x + 6;

$$h := x \rightarrow x^2 + 4\,x + 6 \quad (5)$$

> $factor(g(x));$

$$x^2 - 2 \quad \textbf{(6)}$$

> *factor*(*h*(*x*));

$$x^2 + 4x + 6 \quad \textbf{(7)}$$

The computer essentially solves the quadratic equation to get these answers. It will give you irrational answers with square root signs and imaginary answers with *I* . *I* is just the square root of negative one. It is not a real number. Try the solve command on g(x) and h(x) to see why Maple couldn't factor them. Write a comment about their solutions by backspacing in front of the prompt.

> *solve*(*g*(*x*) = 0, *x*); *Irrational solutions*

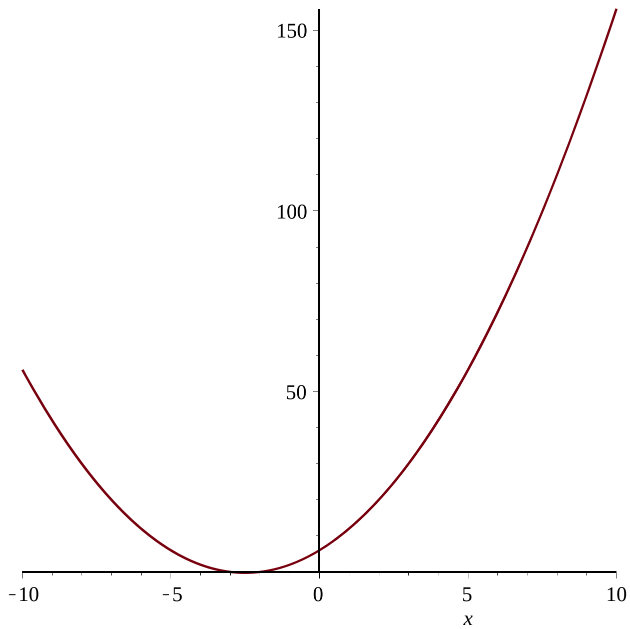
$$\sqrt{2}, -\sqrt{2} \quad \textbf{(8)}$$

> *solve*(*h*(*x*) = 0, *x*); *complex solutions*

$$-2 + \mathrm{I}\sqrt{2}, -2 - \mathrm{I}\sqrt{2} \quad \textbf{(9)}$$

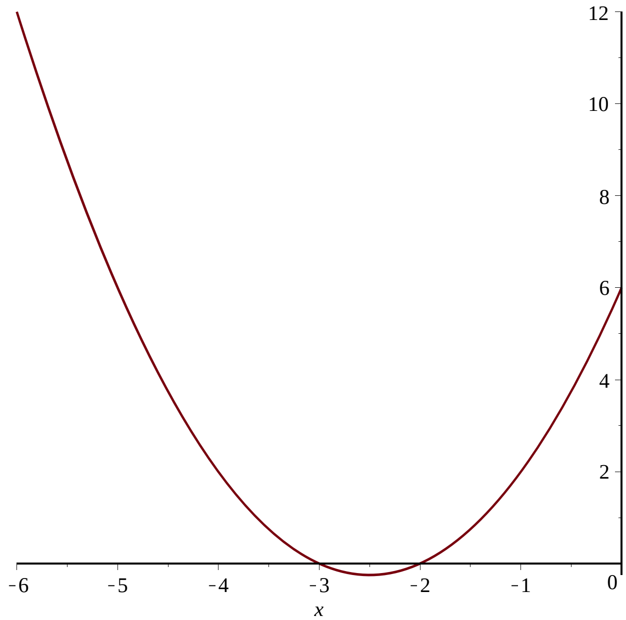
Now try graphing f(x), g(x) and h(x) using the command:

```
> plot(f(x), x=-10..10);
```



You can focus the graph near where it crosses the x axis by focussing on x in [-6,0]

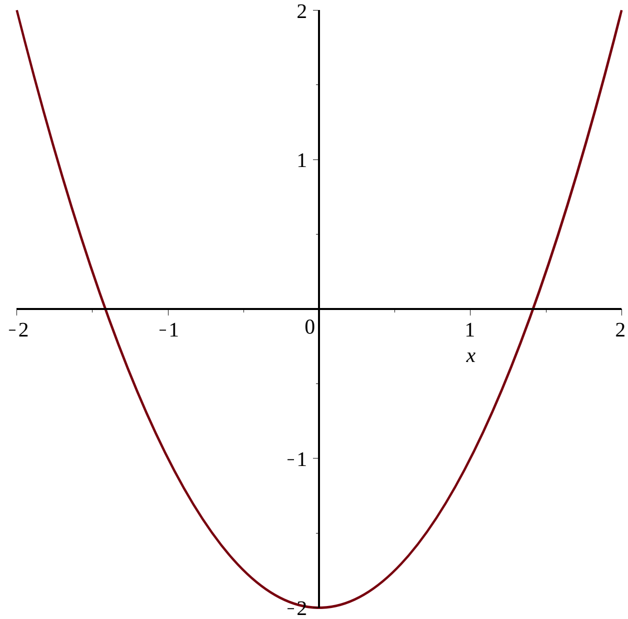
```
> plot(f(x), x=-6..0);
```



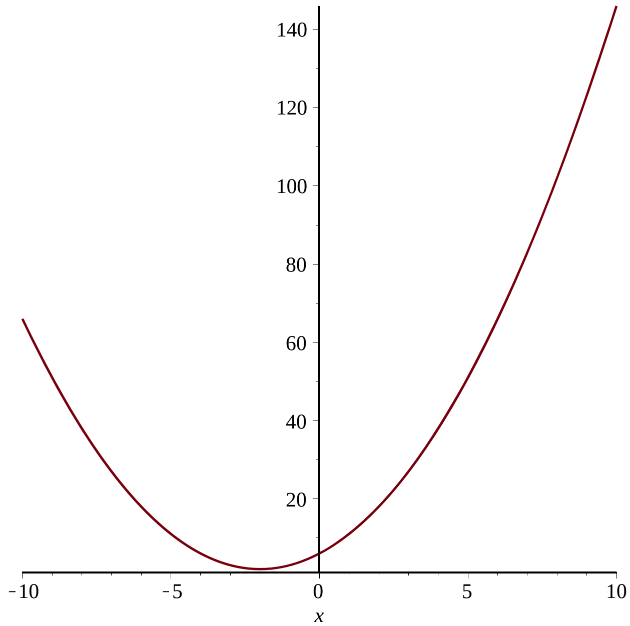
Does it cross at the correct places? YES Write a comment.

Now plot g(x) and h(x) and write comments about the places where they cross the x axis.

> $plot(g(x), x=-2..2);$



> $plot(h(x), x = -10..10);$

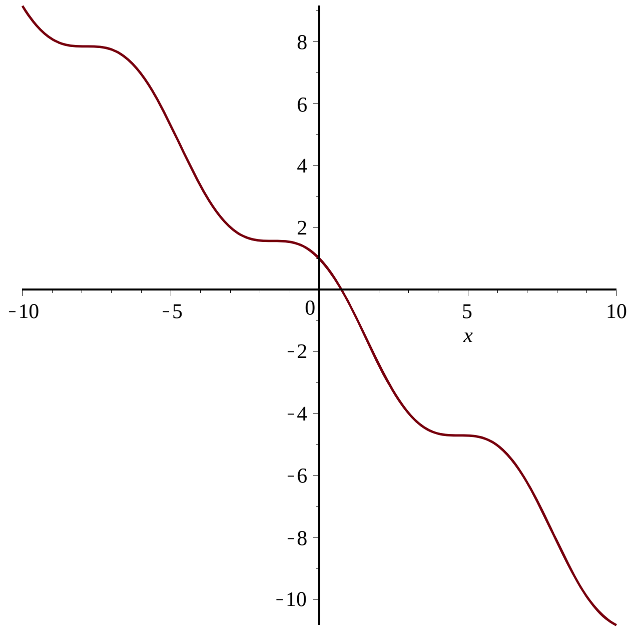


Problem 2: Solving more complicated problems

Remember to save your file and type restart;

Define a function f(x)=cos(x)-x and try to solve f(x)=0. It cannot be factored because it is not a polynomial and the solve command doesn't work. However, you can graph the function, so try to graph it.

```
> plot(cos(x)-x, x=-10..10);
```



> $f := x \rightarrow \cos(x) - x;$

$$f := x \rightarrow \cos(x) - x \quad \textbf{(10)}$$

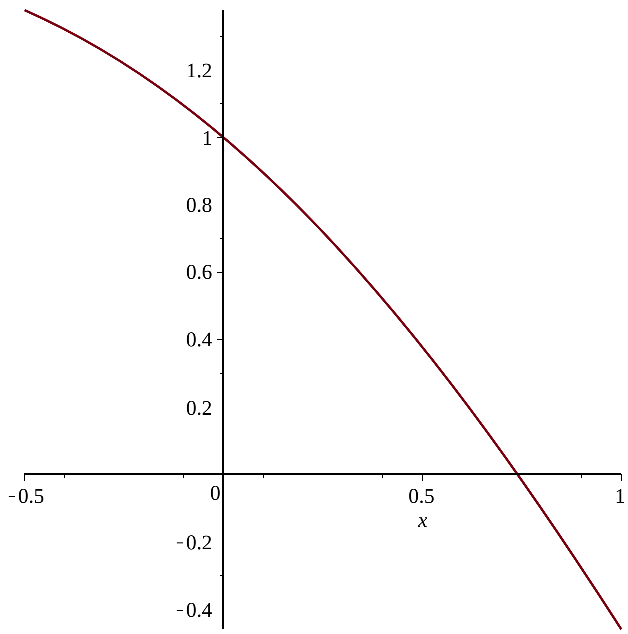
This graph appears to cross the x axis only once and it appears to cross near x=1. It definitely crosses the axis somewhere between 0 and 2. So we can try the fsolve command. This command will try alot of numbers between 0 and 2 until it finds one that solves f(x)=0. However, it will only look for one answer and then quit even if there are two answers. Type:

```
> fsolve(f(x)=0, x=0..2);
```

$$0.7390851332 \quad \textbf{(11)}$$

Now graph the picture again very close to the answer say x in .5 to 1 and see how it crosses the axis. <u>Can you be sure that the function only crosses the axis once?</u> Maybe it crosses again near x=100. <u>One way to be sure it only crosses once would be to show that the function is decreasing</u> (that its graph goes down from left to right). Later in the semester, we will be able to prove that f(x)=cos(x)-x is a decreasing function.

> $plot(\cos(x) - x, x = -.5..1);$



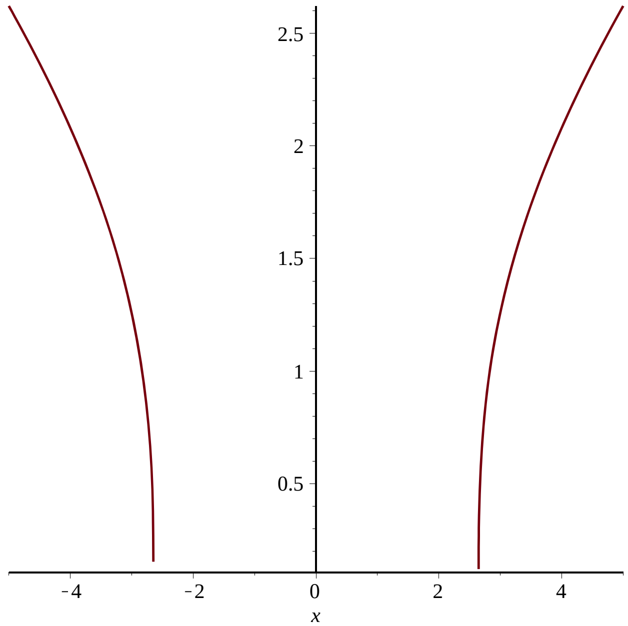

```
>
```

Try to find a solution to g(x)=(x^2-7)^(1/3) by graphing and using fsolve. You will have to find more than one solution by focusing your graph on two different locations and using fsolve twice. Then check your answer by graphing again.

```
> g:=x->(x^2-7)^(1/3);
```

$$g := x \rightarrow \left(x^2 - 7\right)^{1/3} \qquad \textbf{(12)}$$

```
> plot(g(x), x=-5..5);
```



> *fsolve*($g(x) = 0, x = -4..-2$);

$$-2.645751311 \tag{13}$$

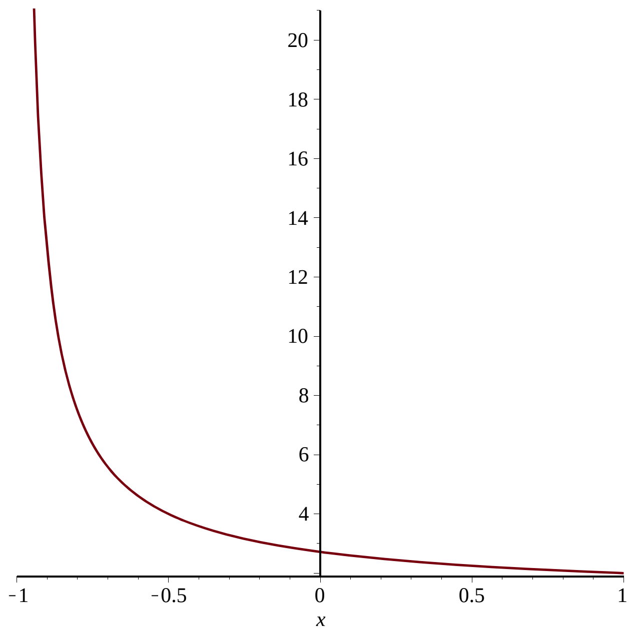
> *fsolve*($g(x) = 0, x = 2..4$);

$$2.645751311 \tag{14}$$

Problem 3: The irrational number e:

Let f(x)=(1+x)^(1/x). Notice that this function is undefined at two numbers, x=-1, and x=0. When x=-1 we get f(-1)=(0)^(-1)=1/0 and at x=0 we get f(0)=(1)^(1/0). Any number divided by 0 is undefined. f(x) is also undefined at x=-2 because f(-2)=(-1)^(-1/2)= 1/ (-1)^(1/2) and the square root of negative one is not a real number. If x>-1 we don't have to worry about having roots of negative numbers. So we can say that the domain of f is (-1,0) u (0, infinity). What does the function look like near x=0 and x=-1? Try the command:

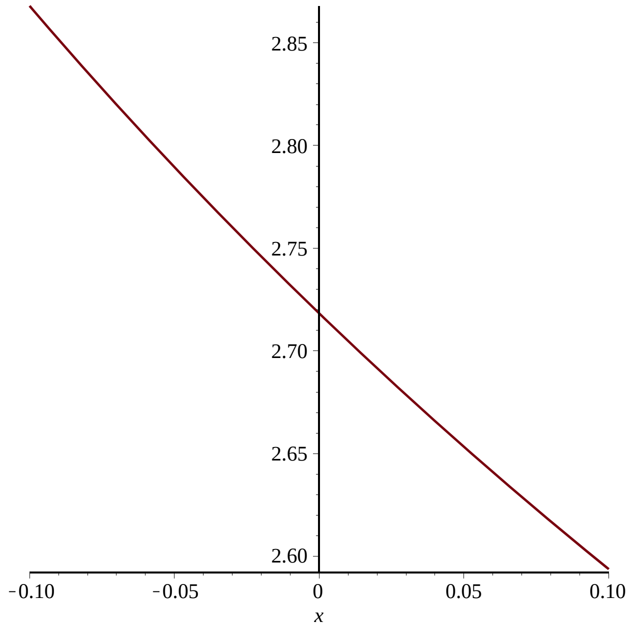
```
> plot((1+x)^(1/x), x=-1...1);
```



f(x) is going to infinity near x=-1. This is called a vertical asmptote. However, near x=0, the function's graph appears to be perfectly nice. You might even think that f(x)=(1+x)^(1/x) is defined at zero.

Suppose you say that 1/0=infinity. Then f(0)=(1+0)^(infinity)=1^(infinity)=infinity.   This doesn't make sense. It doesn't help you guess where the graph appears to cross the x axis. Instead you must focus on x values near 0 and see where it appears to cross. Focus the graph very close to x=0. Try plotting it from x=-0.1 to 0.1. Write a comment estimating the number where the graph appears to cross.

```
> plot( (1 + x)^(1/x), x=-0.1..0.1);
```



Now plot the graph from x=-0.01 to 0.01. *Remember you can cut and paste your plot command onto a new line and just edit the x values.* Then write a comment telling where it appears to cross. What is the largest y value on the graph and what is the smallest?

Now plot it from x=-0.001 to 0.001 and write down where it appears to cross. What is the largest y value on the graph and what is the smallest?

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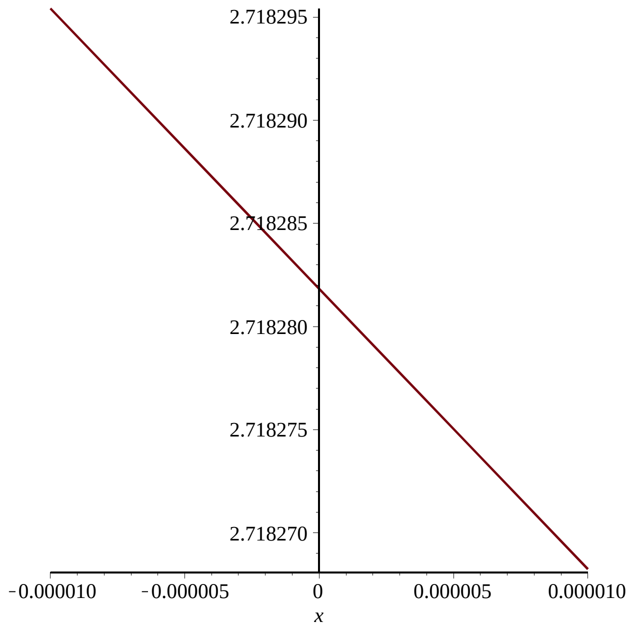
<u>Even though f(0) is not defined, we know that as x gets closer and closer to 0, f(x) gets closer and closer to some number.</u>
You already know an approximate value for this number. It turns out that it is an irrational number. The number is called **e.** It turrns out that e is as important a number as Pi and sqrt(2) and is used in many applications of mathematics. We will see it a lot in Calculus. **We say that the limit of f(x) as x approaches 0 is e.**

```
> plot((1+x)^(1/x), x=-0.00001..0.00001);
```



Exploration:

A: Solving Inequalities.

Let f(x)=x^2 -3x+2 and solve for f(x) >0. To do this solve(f(x)=0) and then graph the function. Find the x values where the graph of f is above the axis.

Let g(x)= sin(x)-x^2+3 and solve for g(x) less than or equal to zero.

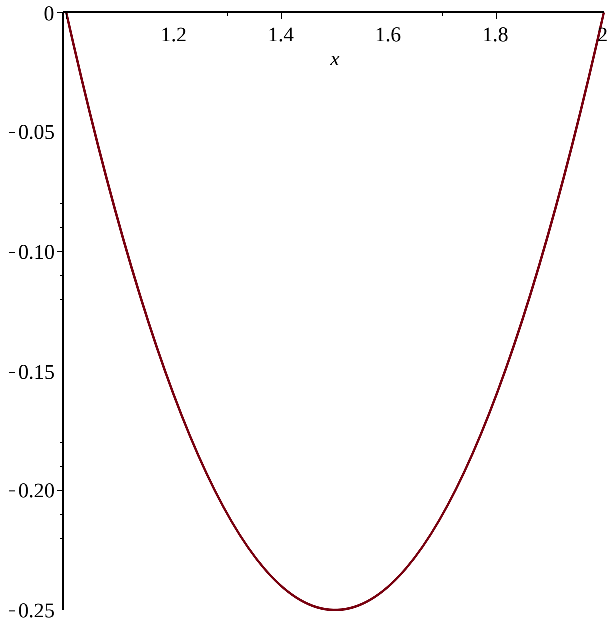
> $f := x \rightarrow x\char`^2\ -3\,x+2;$

$$f := x \rightarrow x^2 - 3\,x + 2 \tag{15}$$

> $solve(\,f(x)\,=0, x);$

$$2, 1 \tag{16}$$

> $plot(\,f(x), x=1\,..2);$



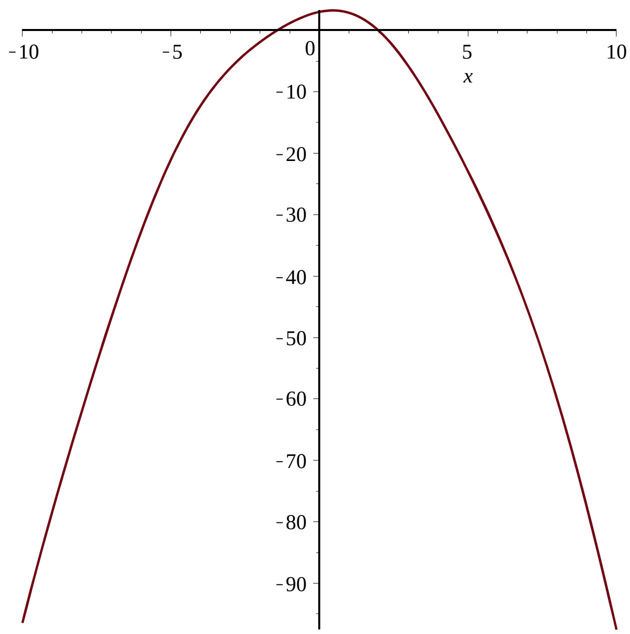
>

So the answer is x< 1 or x>2

> $g := x \to \sin(x) - x\text{^}2 + 3;$

$$g := x \to \sin(x) - x^2 + 3 \tag{17}$$

> $plot(g(x), x = -10 \,..\, 10);$



> $fsolve(g(x) = 0, x = 0 ..5);$

$$1.979320147 \tag{18}$$

> $fsolve(g(x) = 0, x = -5 ..0);$

$$-1.418310092 \tag{19}$$

So the answer is x< -1.41.... or x>1.97...

B: Another limit:

Let f(x)=(x^5-32)/(x-2). Discuss where it is defined and check out the limit where it is not defined.

> $f := x \rightarrow (x\text{^}5-32) / (x-2);$

$$f := x \rightarrow \frac{x^5 - 32}{x - 2} \tag{20}$$

> $factor(f(x));$

$$x^4 + 2x^3 + 4x^2 + 8x + 16 \tag{21}$$

> $g := x \rightarrow x^4 + 2x^3 + 4x^2 + 8x + 16;$

$$g := x \rightarrow x^4 + 2x^3 + 4x^2 + 8x + 16 \tag{22}$$

```
> evalf(g(2));
```

$$80. \tag{23}$$

```
>
```