MAT 155 Project IV Spring 2014

Inverse Functions

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If you cannot recall a command from a previous lab you may **consult the command index** which can be opened up as a second window. The name of the file with the command index is 155.00.00.html.

Do not work on this file. This is just the list of problems. **Open your own file** by selecting "File" on the Maple V bar and choosing "New". Save the new file as **yourname4.mws** and as **yourname4b.mws** regularly using the "save as" command. **Be careful not to save this file with that name or you will overwrite your work!** This window should always have 155.00.04.mws printed on the top.

Do not hit return on this file or the computer will remember it when working on your file and may overwrite variables and functions that you've defined.

Sign your name as a comment at the top of your file by backspacing in front of the prompt and typing it in. Also write Project IV and the names of any other students who are working with you.

At the start of each problem in your file, backspace in front of the prompt and type the Problem Number 1 or Problem Number 2 etc. Then hit return and on the first line with a prompt type

```
> restart;
```

This will clear all previous work.

If you fix or change a line you must go back up to the restart to clear your previous work and then hit enter on all the lines after to get the computer to redo the computations correctly.

Do all the problems in order. Ask a student or the teacher if you are having trouble.

Problem 1: Review:

Define f(x)=sin(x)/x using the command

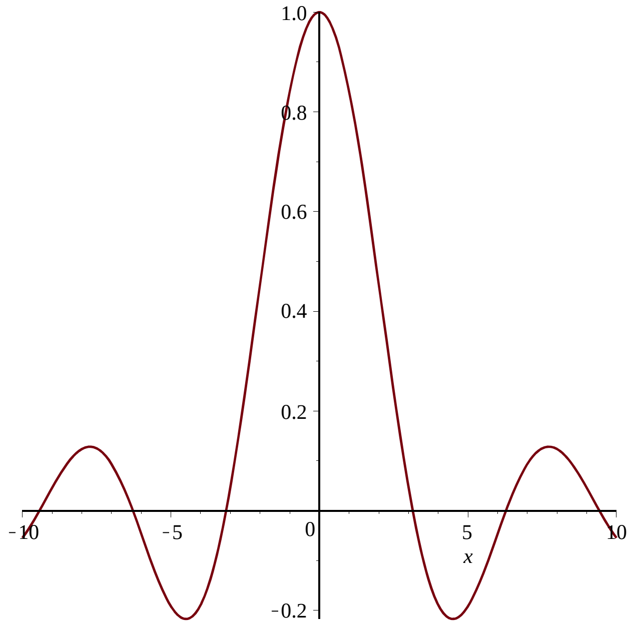
```
> f:= x -> sin(x)/x;
```

$$f := x \rightarrow \frac{\sin(x)}{x} \tag{1}$$

Notice that this function is defined everywhere except where x=0. At x=0, we get f(0)=0/0 which is not defined.

Now you can plot f(x) by typing.

```
> plot(f(x), x=-10..10);
```



Notice that the graph crosses the x axis somewhere between 2 and 4. To get a decimal approximation of the crossing point type:

```
> fsolve(f(x)=0, x=2..4);
```

3.141592654 **(2)**

Note that this is Pi. Recall that sin(Pi)=0 because in calculus we use radians not degrees. Now find out the next number where the graph crosses the x axis.

> $fsolve(f(x) = 0, x = 5..10);$

6.283185307 **(3)**

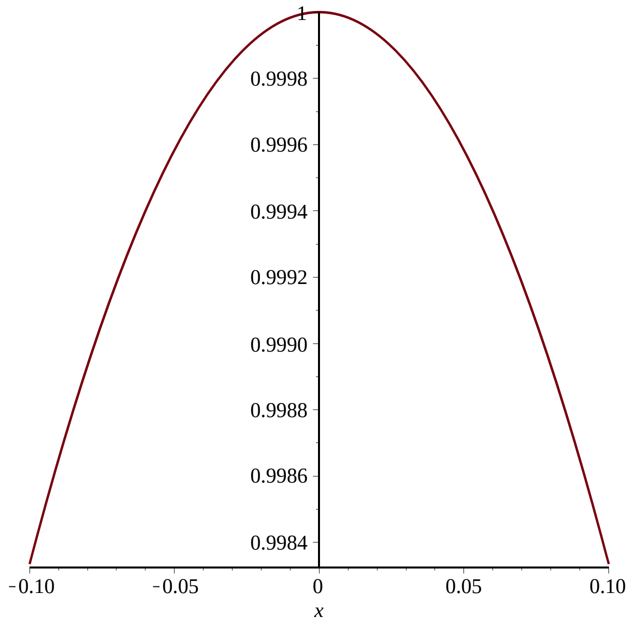
Recall that fsolve command gives only one root in the given interval.

> $fsolve(f(x) = 0, x = 2..10);$

6.283185307 **(4)**

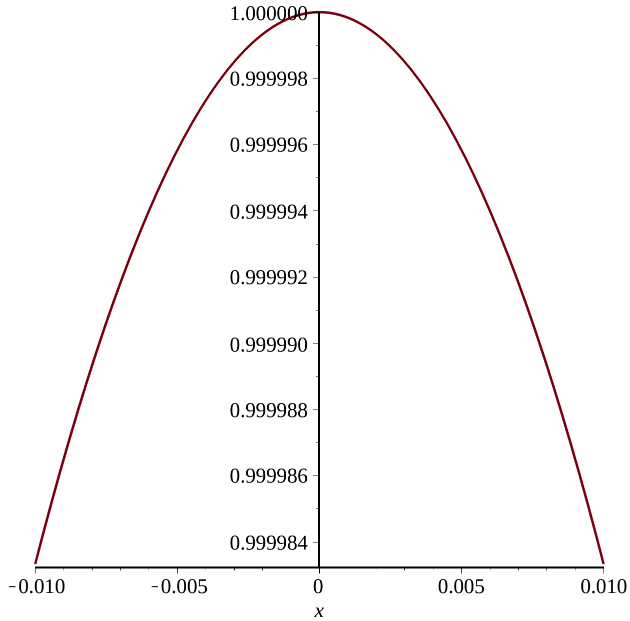
Notice that near x=0, the function's graph appears to be perfectly nice. You might even think that f(x) is defined at zero. Look at x values near 0 and see where it appears to cross. Focus the graph very close to x=0. Try plotting it from x=-0.1 to 0.1. Write a comment estimating the number where the graph appears to cross.

> **plot**$(f(x), x = -.1 ...1);$



Now plot the graph from x=-0.01 to 0.01. *Remember you can cut and paste your plot command onto a new line and just edit the x values.* Then write a comment telling where it appears to cross. What is the largest y value on the graph and what is the smallest?

> **plot**$(f(x), x=-.01 ...01)$;



Again it looks like the graph crosses (0,1).

It appears that as x gets closer and closer to zero, the graph gets closer and closer to 1. We say that the **limit of f(x) as x approaches 0 is 1** and write $\lim_{x \to 0} f(x) = 1$**.**

Problem 2: Inverses and the horizontal line test:
Sometimes you don't just want to solve f(x)=0. Suppose you want to solve f(x)=2 and f(x)=4 and f(x) = 15 and so on.

Rather than solving the problem repeatedly, it is easier to find an inverse function, f^{-1} = invf(This is just a name), which will just cancel everything f does, so that you need only evaluate invf(2), invf(4) and invf(15) and so on. For example, if f(x)=x+2, then invf(x)=x-2 and if f(x)=x^3 then invf(x)=x^(1/3).

To get Maple to find an inverse for you, you just solve f(y)=x for y *(you may copy and paste the commands into your file):*

```
> f:= x -> x^3;
```

$$f := x \to x^3 \tag{5}$$

```
> solve(f(y)=x,y);
```

$$x^{1/3}, -\frac{1}{2} x^{1/3} + \frac{1}{2} \mathrm{I} \sqrt{3}\, x^{1/3}, -\frac{1}{2} x^{1/3} - \frac{1}{2} \mathrm{I} \sqrt{3}\, x^{1/3} \tag{6}$$

You will notice that Maple gives 3 answers but 2 are imaginary. So the only real solution is y=x^(1/3). So we define:

```
> invf:=x->x^(1/3);
```

$$invf := x \rightarrow x^{1/3} \quad (7)$$

Suppose you want to find the inverse of x^2? Let g(x)=x^2 and solve g(y)=x for y. You will get two answers, so there is no inverse for g(x)!

> $f := x \rightarrow x$^2;

$$f := x \rightarrow x^2 \quad (8)$$

> $solve(\,f(y) = x, y\,)$;

$$\sqrt{x},\ -\sqrt{x} \quad (9)$$

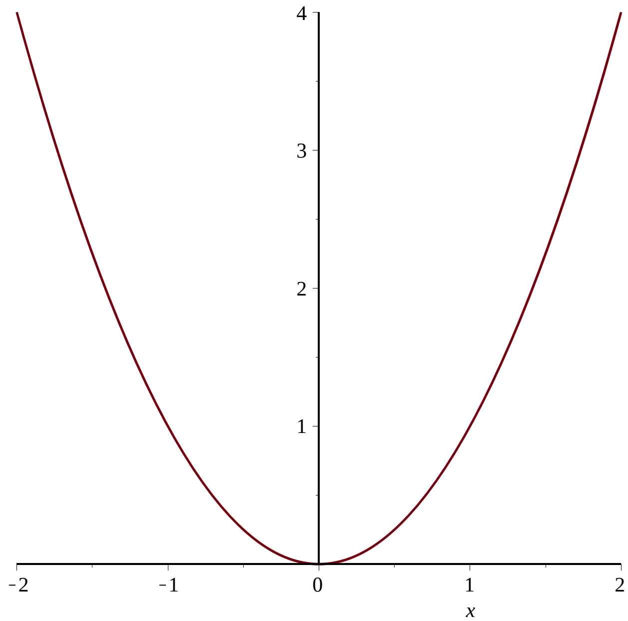
If you graph a function f(x) you can tell whether it has an inverse or not. It has an inverse if for every y value, you can draw a horizontal line through y and find exactly one x value such that f(x)=y. That is the horizontal line can only cross the graph once.
Use the plot command to check if f(x)=x^3 and g(x)=x^2 satisfy the horizontal line test.

Then let h(x)=2^x and graph it to see if it has an inverse.

> $f := x \rightarrow x$^2;

$$f := x \rightarrow x^2 \quad (10)$$

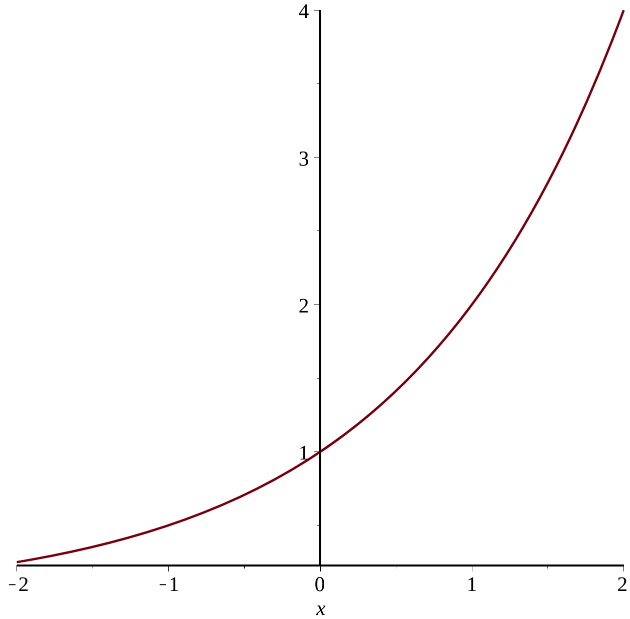
> **plot**$(\,f(x), x = -2..2\,)$;



> h := $x \rightarrow 2$^x;

$$h := x \rightarrow 2^x \qquad \textbf{(11)}$$

> **plot**$(h(x), x = -2..2)$;



This h(x) has an inverse!

Use the solve command to try to find the inverse of h(x). You will get a strange fraction with the function ln(x) in it. ln(x) is called the natural log function. It is in fact the inverse of e^x. The natural log function is used to find inverses of exponential functions. Remember that to solve x=(5y)^3 for y you take the cube root of both sides of the equation. So if x=2^y, you take the natural log of both sides ln(x)=ln(2^y). Use Maple to find ln(x)=ln(2^y)=yln(2), so y=ln(x)/ln(2) (see P.6 Example 6).

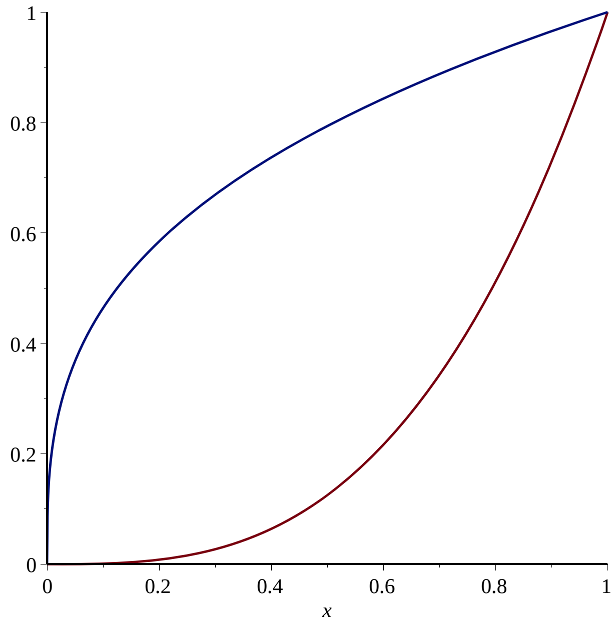
```
> solve(h(y) = x, y);
```

$$\frac{\ln(x)}{\ln(2)} \tag{12}$$

Problem 3: graphing functions and inverses,

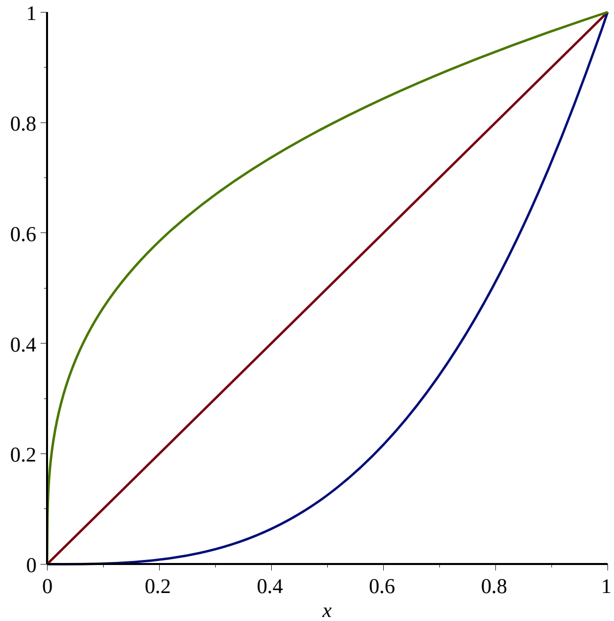
Let f(x)=x^3 again and invf(x)=x^(1/3). Graph them together using the command (*You may wish to copy and paste it).*

```
> plot({x^3, x^(1/3)}, x=0..1);
```



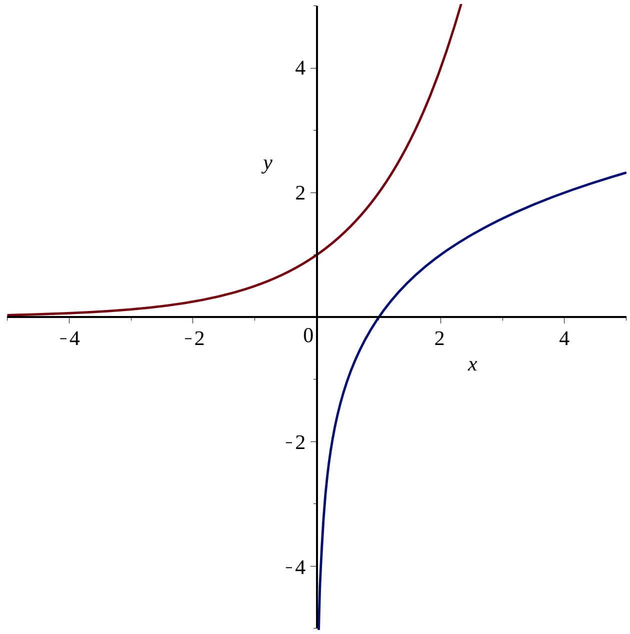
You may notice that the graphs have an interesting symmetry around the y=x line:

```
> plot({x^3,x, x^(1/3)}, x=0..1);
```



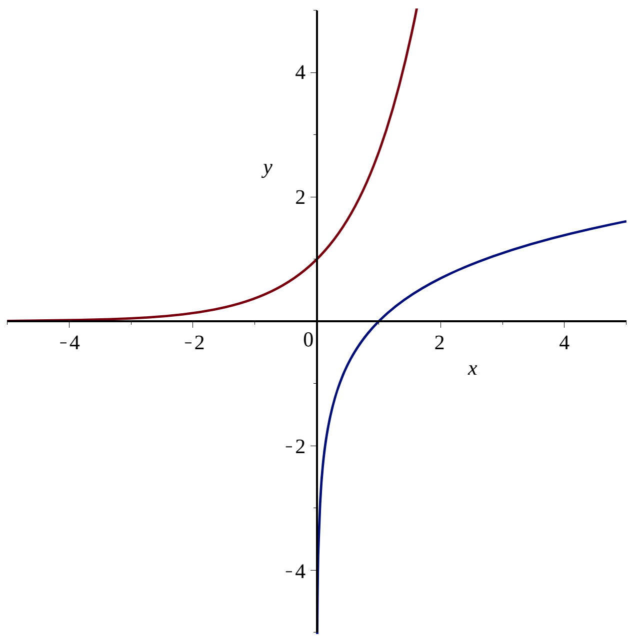
Why does this happen? Compare with other functions like h(x) and invh(x) from problem 2 using the command:

```
> plot({2^x, ln(x)/ln(2)}, x=-5..5, y=-5..5);
```



Compare exp(x), which is Maple's special way of taking e^x, and ln(x) which is its inverse. Use the x= and y= parts of the command to make a nice square plot.

```
> plot({exp(x), ln(x)}, x=-5..5, y=-5..5);
```



Now define g(x) to be the function which describes a line of slope 3 through (1,2) and solve for its inverse and graph them together.

> $g := x \rightarrow 3(x-1)+2;$

$$g := x \rightarrow 3\,x - 1 \tag{13}$$

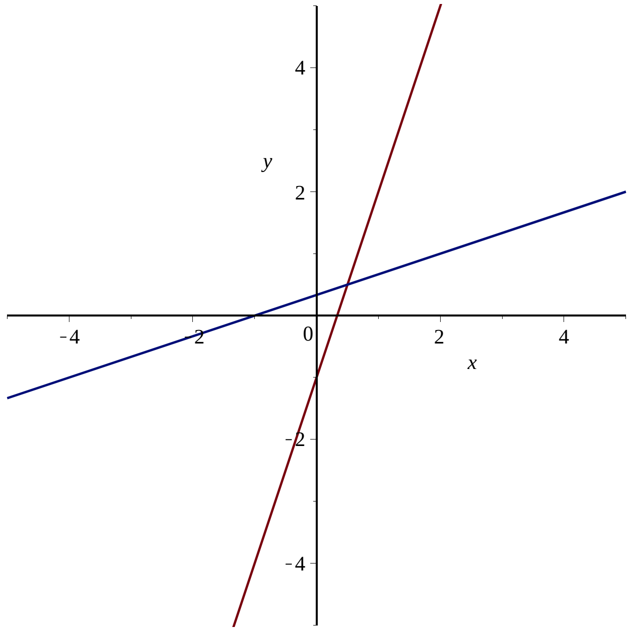
> $solve(g(y) = x, y);$

$$\frac{1}{3}\,x + \frac{1}{3} \tag{14}$$

> $invg := x \rightarrow \frac{1}{3}\,x + \frac{1}{3};$

$$invg := x \rightarrow \frac{1}{3}\,x + \frac{1}{3} \tag{15}$$

> $plot(\{g(x), invg(x)\}, x = -5..5, y = -5..5);$



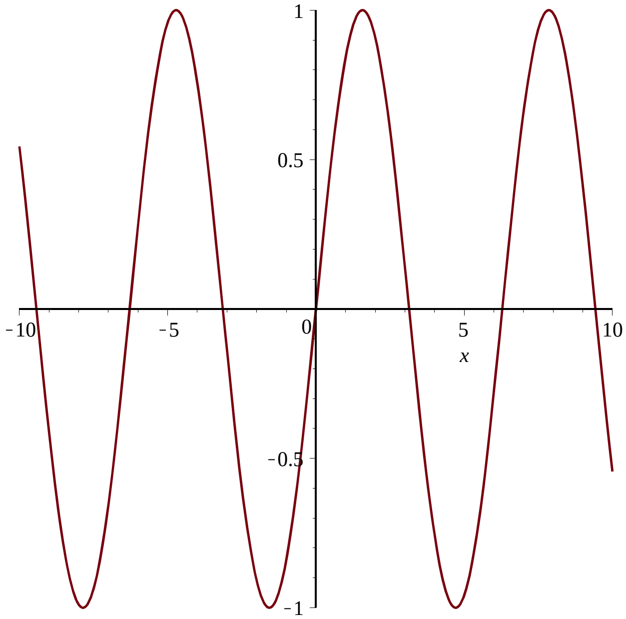
Exercises 21 through 25 on page 44 of the text.
Try instead 4/e p.45 #37--#42 (5/e p.44 #29--#36) Some of problems will be taken as exam problems among these.

Exploration:
A: If you graph f(x)=cos(x) on [-10,10], you will see that it fails the horizontal line test. However, it passes the horizontal line test if you graph f(x)=cos(x) on [0, Pi]. So we could define an inverse for this restricted definition of cosine. It is called arccos(x). Now graph sin(x) on [-10,10]. Choose a restricted domain for sin(x) that includes x=0 and passes the horizontal line test.

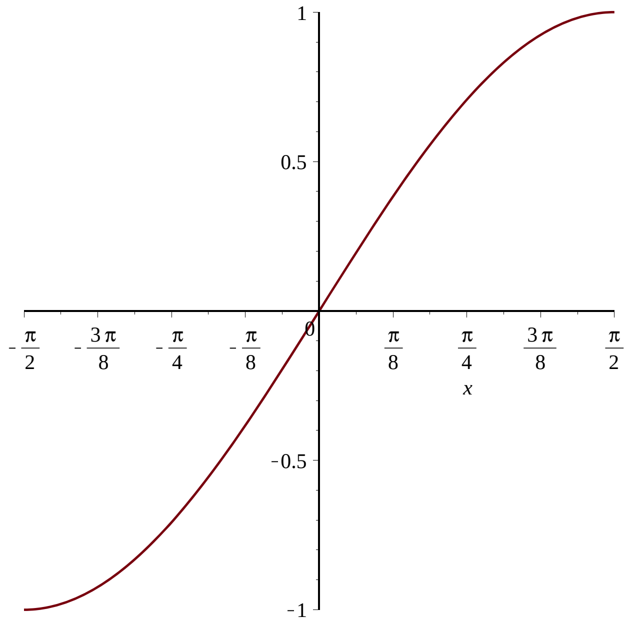
> $f := x \rightarrow \sin(x);$

$$f := x \rightarrow \sin(x) \quad \textbf{(16)}$$

> $plot(\,f(x), x = -10\,..10);$



> $plot\left(f(x), x=-\frac{\mathrm{Pi}}{2} .. \frac{\mathrm{Pi}}{2}\right);$

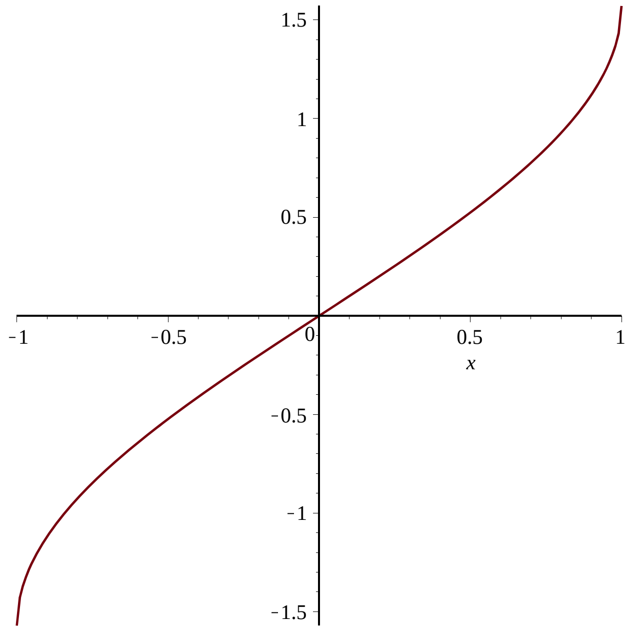


> $solve(f(y) = x, y);$

$$\arcsin(x) \tag{17}$$

>

> $plot(\arcsin(x), x = -1..1);$


1.5
1
0.5
-0.5
-1
-1.5
-1
-0.5
0
0.5
1
x

>