MAT 155 Project V Spring 2014

Limits and Vertical Asymptotes

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Do not work on this file. This is just the list of problems. Open your own file and save it regularly as yourname5.mws and as yourname5b.mws. Sign your name as a comment at the top of your file by backspacing in front of the prompt and typing. Also write Project V and the names of any other students you are working with. Don't forget to number your problems and to type restart at the beginning of each problem.

If you cannot complete a problem, go on to the next one and return to the problem later. Just be sure to keep all your **problems in order** on the file. You can **get a new prompt** by selecting the prompt button right below the word "spreadsheet". It has the symbol "[>" on it. You must hit enter on every line of the problem in order, including the restart line, to review what you've done for the Maple program.

If you cannot recall a command from a previous lab you may consult the command index which can be opened up as a second window. The name of the file with the command index is 155.00.00.html.

Problem 1: Computing Limits Analytically: To compute the following limit: $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

a) First define the function f(x) = $\frac{x^2 - 9}{x - 3}$

Type in the command and be careful to use parenthesis correctly! Consult the command index if necessary.

$> f := x \to \frac{(x^2 - 9)}{(x - 3)};$

$$x \to \frac{x^2 - 9}{x - 3} \quad (1)$$

b) Check what happens if you try to compute f(3).

$> f(3);$

Error, (in f) numeric exception: division by zero

c) Then use the simplify command (see the command index) to simplify f so that and call the simplified version g(x). Note that g(x) agrees with f everywhere except at x=3 where f is undefined. So $\lim_{x \to 3} \left(f(x) = \lim_{x \to 3} g(x) \right)$

$> simplify\left(\frac{x^2 - 9}{x - 3}\right);$

$$x + 3 \quad (2)$$

$> g := x \to x + 3;$

$$x \to x + 3 \quad (3)$$

d) Now find the limit of g(x) using direct substitution: g(3);

e) Finally use the limit command (see the command index)

$> g(3);$

$$6 \quad (4)$$

> limit(f (x), x = 3);

$$6 \tag{5}$$

Problem 2: Computing Limits Analytically: Do steps a-e to compute $\lim_{x \to 3} \frac{x^5 - 3x^4 - 16x^3 + 48x^2}{x^2 - 9}$

> $f := x \to \frac{x^5 - 3x^4 - 16x^3 + 48x^2}{x^2 - 9}$;

$$x \to \frac{x^5 - 3x^4 - 16x^3 + 48x^2}{x^2 - 9} \tag{6}$$

> f(3);

Error, (in f) numeric exception: division by zero

> $simplify\left(\frac{x^5 - 3x^4 - 16x^3 + 48x^2}{x^2 - 9}\right)$;

$$\frac{(x^2 - 16)\, x^2}{x + 3} \tag{7}$$

> $g := x \to \frac{(x^2 - 16)\, x^2}{x + 3}$;

$$x \to \frac{(x^2 - 16)\, x^2}{x + 3} \tag{8}$$

> g(3);

$$-\frac{21}{2} \tag{9}$$

> limit(g(x), x = 3);

$$-\frac{21}{2} \tag{10}$$

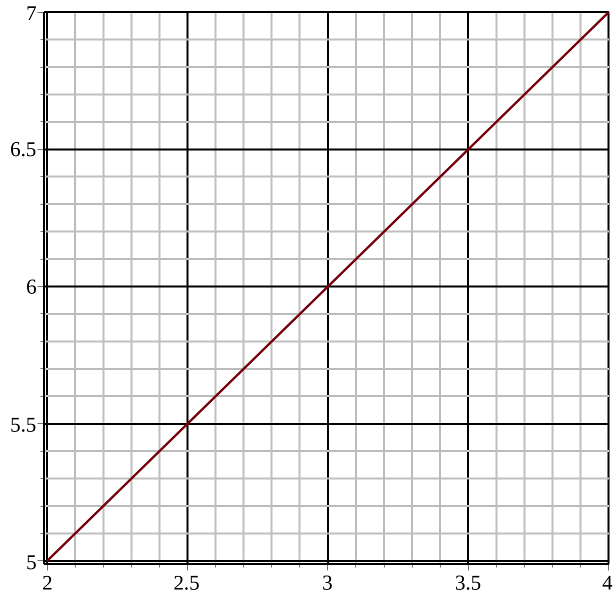
Have you been remembering to save your work?

Problem 3: Verifying Limits Graphically: Go back to problems 1 and 2 and add a step f) plot f(x) for x in [2,4] and confirm your limits graphically. ***Use the*** [> ***button to insert a prompt to add these extra lines.***

> $f := x \to \frac{(x^2 - 9)}{(x - 3)}$;

$$x \to \frac{x^2 - 9}{x - 3} \tag{11}$$

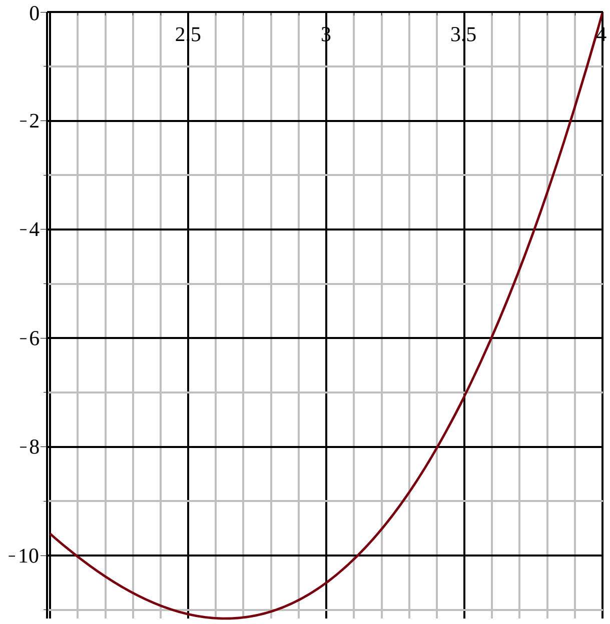
> plot(f (x), x = 2 ..4);



$> f := x \rightarrow \frac{x^5 - 3\,x^4 - 16\,x^3 + 48\,x^2}{x^2 - 9};$

$$x \rightarrow \frac{x^5 - 3\,x^4 - 16\,x^3 + 48\,x^2}{x^2 - 9} \quad \textbf{(12)}$$

$> plot(\,f(x),\, x = 2\,..4\,);$



> ?

Problem 4: A Vertical Asymptote: If we try to use steps a-e from above to compute $\lim_{x \to 3} \frac{x+3}{x^2-9}$

> $f := x \to \frac{x+3}{x^2-9};$

$$x \to \frac{x+3}{x^2-9} \qquad \textbf{(13)}$$

> $f(3);$

```
Error, (in f) numeric exception: division by zero
```

> $simplify\left(\frac{x+3}{x^2-9}\right);$

$$\frac{1}{x-3} \qquad \textbf{(14)}$$

> $g := x \to \frac{1}{x-3};$

$$x \to \frac{1}{x-3} \qquad \textbf{(15)}$$

> $g(3)$;

Error, (in g) numeric exception: division by zero

> $limit(g(x), x = 3)$;

$$undefined \tag{16}$$

then we get a problem. When we try to use direct substitution, g(3) it still has a division by 0 error. When we use the limit command we are given the answer undefined. If we graph the function for x in [2,4] we also have trouble, so try the plot command with y values as well as x values:

```
> plot(f(x), x=2..4, y=-100..100);
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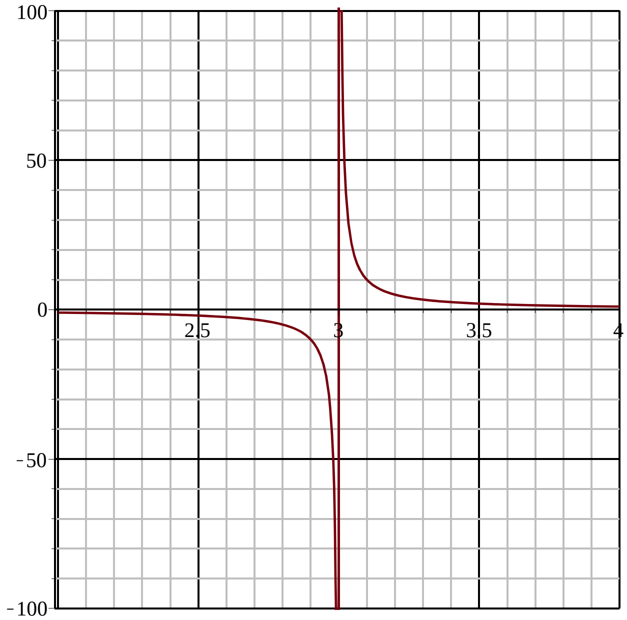


Now we can see that there is a vertical asymptote, so the limit is undefined. We can also say that the limit from the left is - infinity and the limit from the right is positive infinity because the graph goes down to - infinity on the left of the asymptote and comes down from positive infinity on the right of the asymptote.

Repeat this process to compute $\lim_{x \to 3} \frac{x^2 - 16}{(x-3)^2}$

> $f := x \to \frac{x^2 - 16}{(x-3)^2}$;

$$x \to \frac{x^2 - 16}{(x-3)^2} \tag{17}$$

> $f(3);$

```
Error, (in f) numeric exception: division by zero
```

> $simplify\left(\frac{x^2-16}{(x-3)^2}\right);$

$$\frac{x^2-16}{(x-3)^2} \tag{18}$$

> $g := x \to \frac{x^2-16}{(x-3)^2};$

$$x \to \frac{x^2-16}{(x-3)^2} \tag{19}$$

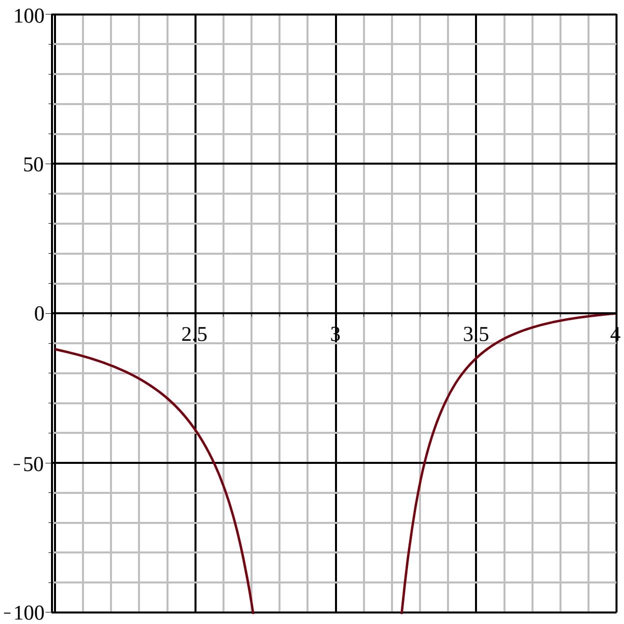
> $g(3);$

```
Error, (in g) numeric exception: division by zero
```

> $limit(g(x), x=3);$

$$-\infty \tag{20}$$

> $plot(\, f(x), x=2..4, y=-100..100);$



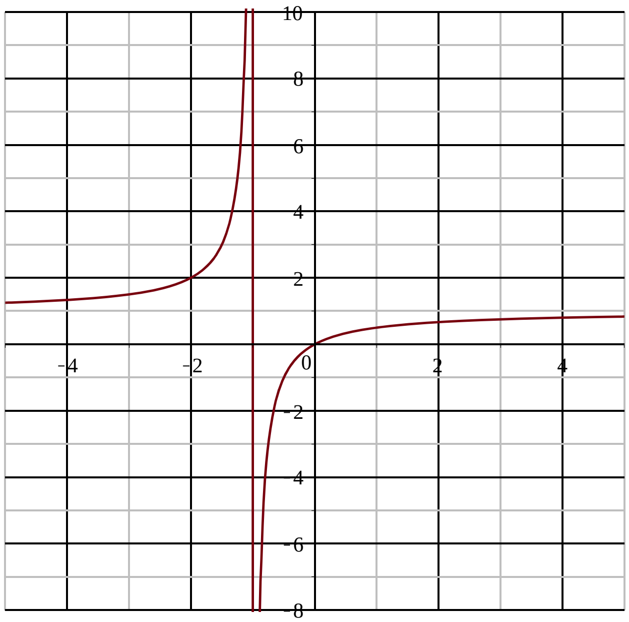
Before going on to the exploration be sure that you have done all the problems.

Exploration:
A: Graph x/(x+1) and tan(x) and find the vertical asmptotes. Redo the graphs with y bounds.

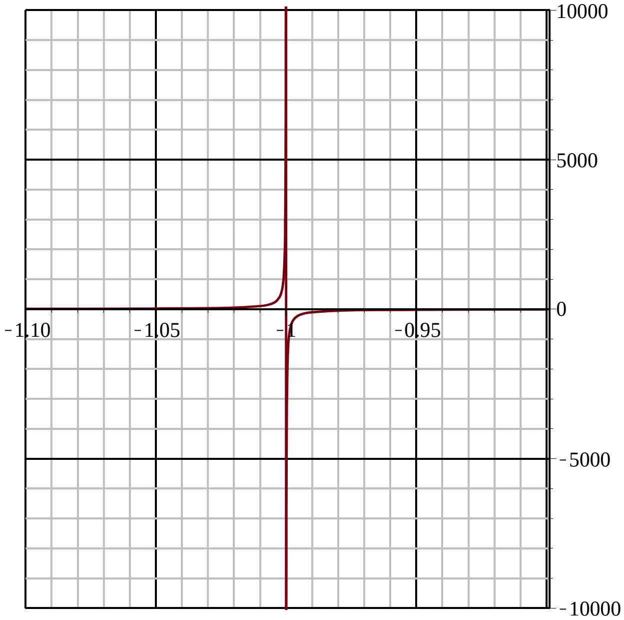
B: Let f(x) = $\frac{x^4+14x^3+71x^2+154x+120}{x^3+6x^2+11x+6}$. Find out the values of x for which this function is undefined. Hint: use the solve command. Then find the limit as x approaches each of these three values both analytically
and graphically. Hint: for some of the graphs you may need y bounds.

A. Vertical asymptotes are where the denominators are vanishing. In case of x/(x+1), x=-1 is the vertical asymptote and in case of tan(x) , x=nPi/2 for any integer n. Graphically:

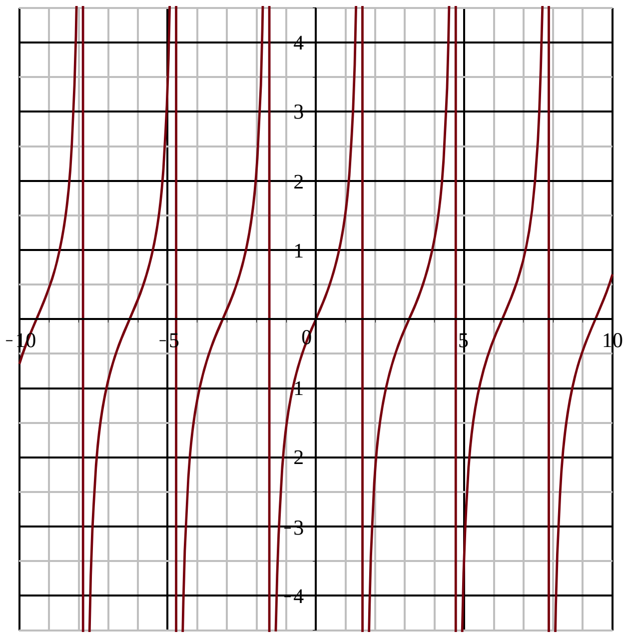
$> plot\left(\frac{x}{(x+1)}, x=-5..5\right);$



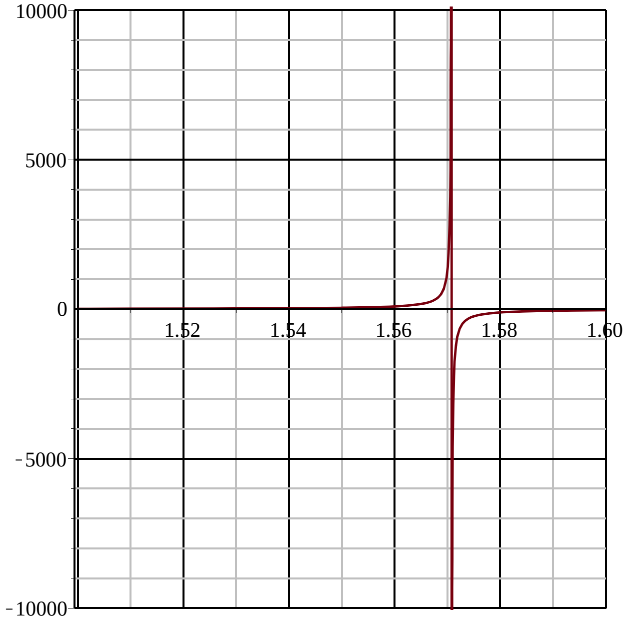
$> plot\left(\frac{x}{(x+1)}, x=-1.1..-0.9, y=-10000..10000\right);$



> $plot(\tan(x), x = -10..10);$



> $plot(\tan(x), x = 1.5..1.6, y = -10000..10000);$



B. The function is not defined at which the denominator is vanishing:

> solve(x^3 + 6 * x^2 + 11 * x + 6 = 0, x);

$$-3,\ -2,\ -1 \tag{21}$$

We now find limits for these three x-values.

> f := x→ (x^4 + 14 * x^3 + 71 * x^2 + 154 * x + 120) / (x^3 + 6 * x^2 + 11 * x + 6);

$$x \rightarrow \frac{x^4 + 14\,x^3 + 71\,x^2 + 154\,x + 120}{x^3 + 6\,x^2 + 11\,x + 6} \tag{22}$$

> limit(f(x), x =-3);

$$-1 \tag{23}$$

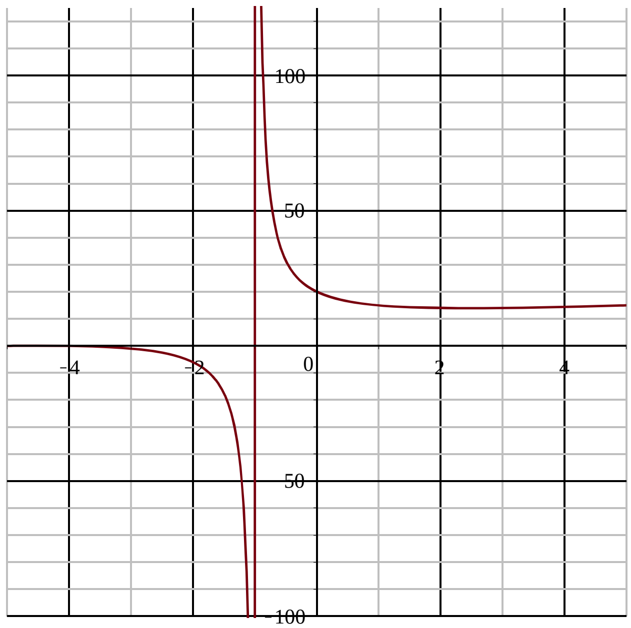
> limit(f(x), x =-2);

$$-6 \tag{24}$$

> limit(f(x), x =-1);

$$\mathit{undefined} \tag{25}$$

> plot(f(x), x =-5 ..5);


100
50
–4
–2
0
2
4
50
–100

> ?