MAT 155 Project VI Spring 2014

Finding Tangent Lines

BYUNG DO PARK

Do not work on this file. This is just the list of problems. Open your own file and save it as yourname6.mws and as yourname6b.mws. Sign your name as a comment at the top of your file by backspacing in front of the prompt and typing. Also write Project VI and the names of any other students you are working with. Don't forget to number your problems and to type restart at the beginning of each problem.

If you cannot complete a problem, go on to the next one and return to the problem later. You can **get a new prompt** by selecting the prompt button right below the word "spreadsheet". It has the symbol "[>" on it. You must hit enter on every line of the problem in order, including the restart line, to review what you've done for the Maple program.

If you cannot recall a command from a previous lab you may consult the command index which can be opened up as a second window. The name of the file with the command index is 155.00.00.html.

Problem 1: Loops: Before beginning to study tangent lines we will learn one of the most fundamental techniques used in computer programming. This technique is called a **loop**. In later problems we will use a loop program to
graph many secant lines to a curve without having to repeatedly type the plot command. To make life a little easier we won't do any calculus in this problem.

Suppose we want the computer to write a list 1^2, 2^2, 3^2...10^2. We tell it that **for j** running **from 1 to 10** it should **do** the command **j^2**. The maple program understands words like **for, from, to** and **do**. Sometimes there is a long list of commands for the computer to complete so we end the list of things it should do with the word **od** as follows:
> for j from 1 to 10 do j^2 od;
This is a one line program called a loop because it repeats the same command over and over. ***Before hitting enter on a loop command like the one below, always save your work. Your computer might crash.***

```
> for j from 1 to 10 do j^2 od;
```

(1)

1
4
9
16
25
36
49
64
81
100

Copy this loop program using copy and paste and try it out on your file. Then write a program which finds j^2 for j running from 1 to 25. Then write a program which finds sin(Pi*j) for j=1 to 5.

```
> for j from 1 to 25 do j^2 od;
```

```
> for j from 1 to 5 do sin(π·j) od;
```

$$0,\ 0,\ 0,\ 0,\ 0 \qquad \textbf{(2)}$$

Problem 2: Plotting Secant Lines: Define f(x)=x^3 using

[> f:= x -> x^3.

Now we want to find the secant line which meets the graph of f(x) above x=2 and x=3. To do this we need to use the point slope formula: y= m(x- a) + b where m is the slope and (a,b) is a point on the line. Since (2, f(2)) is on the line a=2 and b=f(2) will do fine. Use the commands

[> a:=2;

[> b:=f(a);

to set these values on the computer. To find the slope, m, we use the fact that the secant line is passing through (2, f(2)) and (3, f(3)). So

[> m:= (f(3)-f(2))/(3-2);

Now you can plot the secant line with f using the command:

[> plot({f(x), m*(x-a)+f(a)}, x=0..3);

Verify that the secant line does in fact cross the graph of f above x=2 and x=3.

```
> f := x → x^3;
```

$$x \to x^3 \qquad \textbf{(3)}$$

```
> a := 2;
```

$$2 \qquad \textbf{(4)}$$

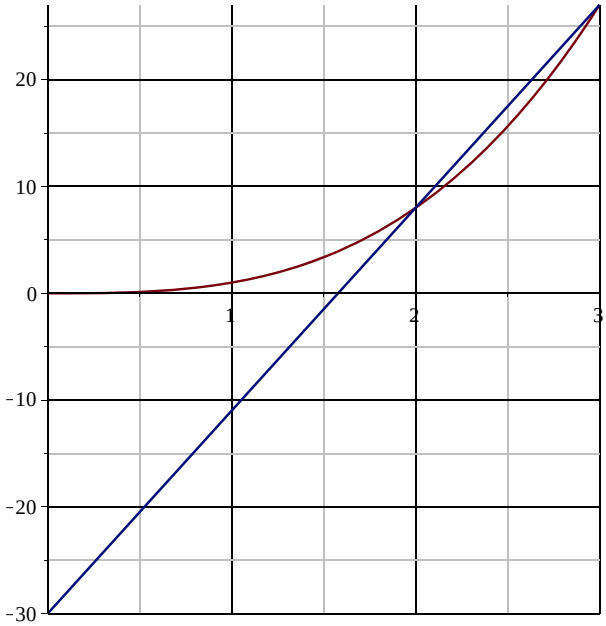
```
> b := f(a);
```

$$8 \qquad \textbf{(5)}$$

```
> m := (f(3)-f(2))/(3-2);
```

$$19 \qquad \textbf{(6)}$$

```
> plot( {f(x), m*(x-a)+f(a)}, x=0..3);
```

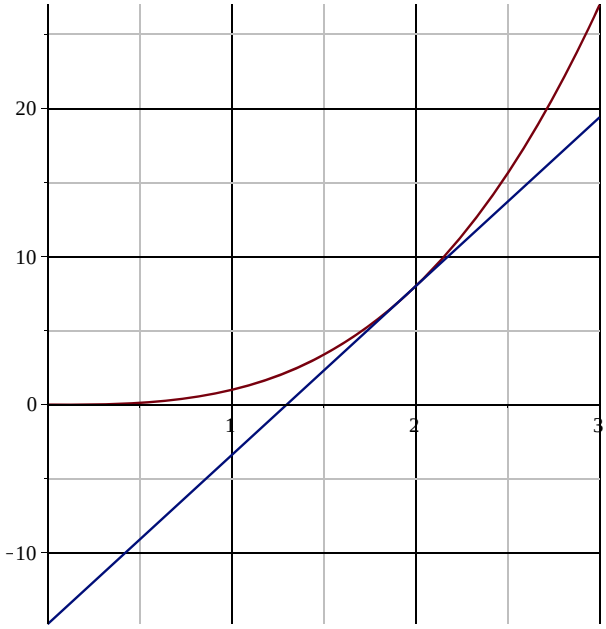


Now find the secant line which meets the graph of f(x)=x^3 above x=2 and x=1.9 .

```
> m := (f(1.9)-f(2))/(1.9-2);
```

11.41 **(7)**

```
> plot({f(x), m*(x-a)+f(a)}, x=0..3);
```

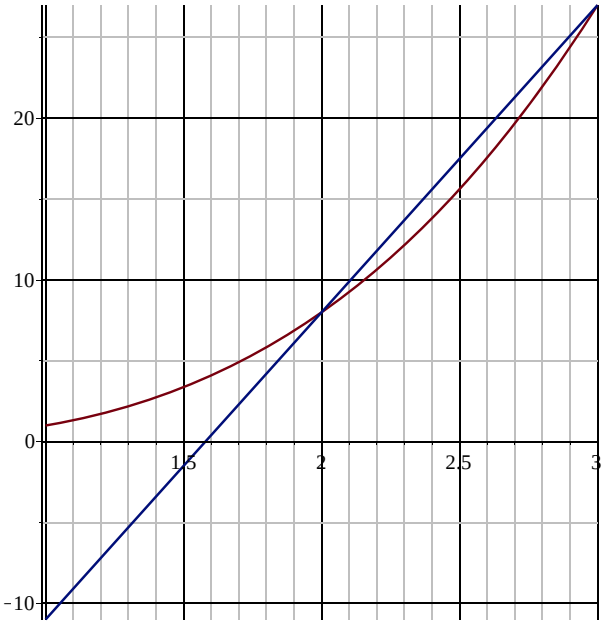


Problem 3: Using Loops to Find and Understand Tangent Lines:

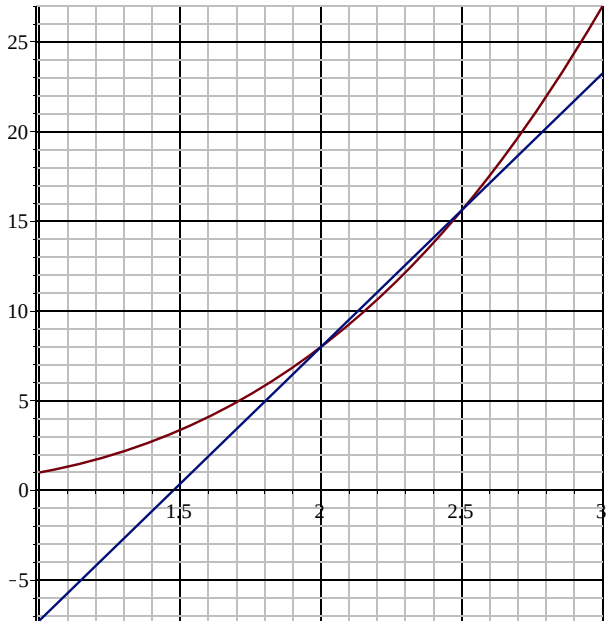
a) To find the tangent line to f(x)=x^3 through the point (2, f(2)), we need to look at many secant lines each crossing the graph of f(x) a a point above 2 and a point near 2. These secant lines will look more and more like the tangent line at 2 when the second point gets very close to 2. For example, we could find the secant lines which cross f(x) above 2 and above (2+1/j) where j is getting larger and larger. This would require a lot of work on our part to plug in different values of j, compute the slope and then plot the curve, so we use a loop.

To compute secant lines between the point 2 and points closer and closer to 2, we tell the computer that for j= 1 to 10 it should do the computation m:=(f(2+1/j)-f(2))/(2+1/j-2) to find the slope and then to plot the secant line. *Save your work* ***and then type the following program without hitting return until after the od is typed.***

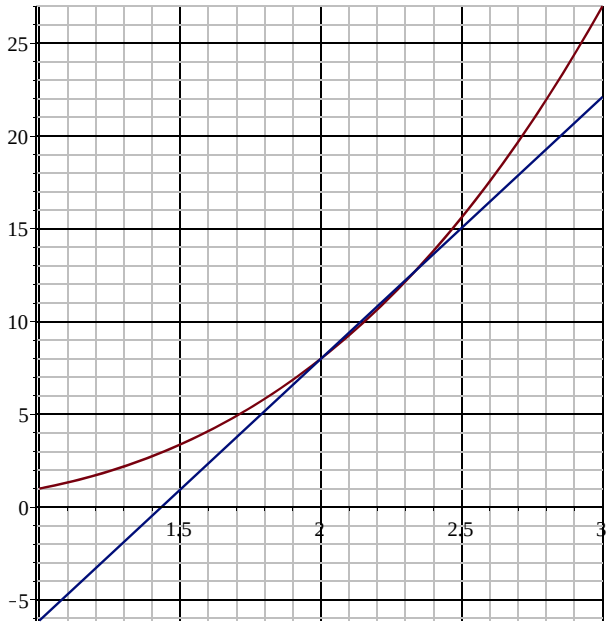
```
> for j from 1 to 10 do m:= (f(2+1/j)-f(2))/(1/j); plot({f(x), m*
  (x-2)+f(2)}, x=1..3); od;
```



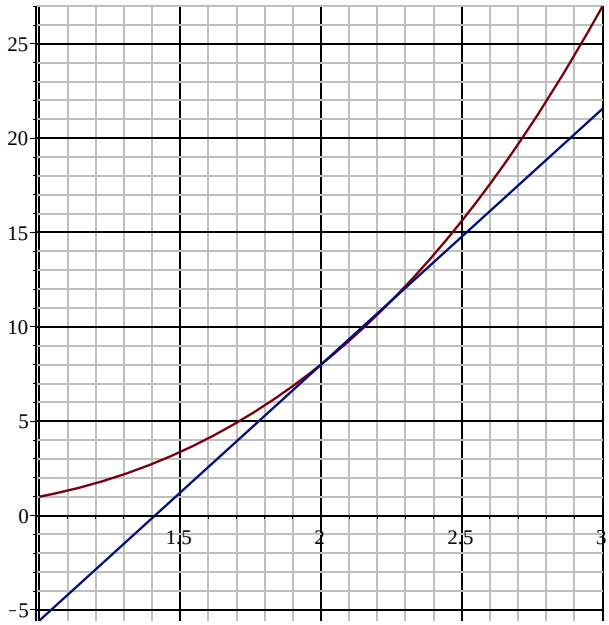
$$\frac{61}{4}$$



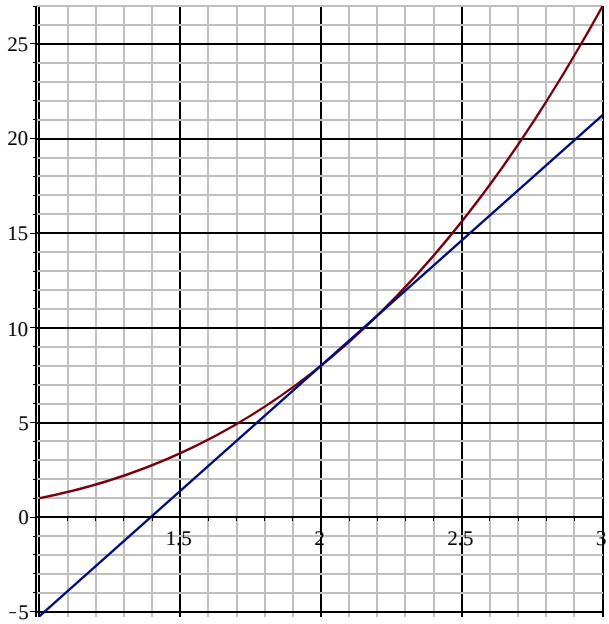
$$\frac{127}{9}$$



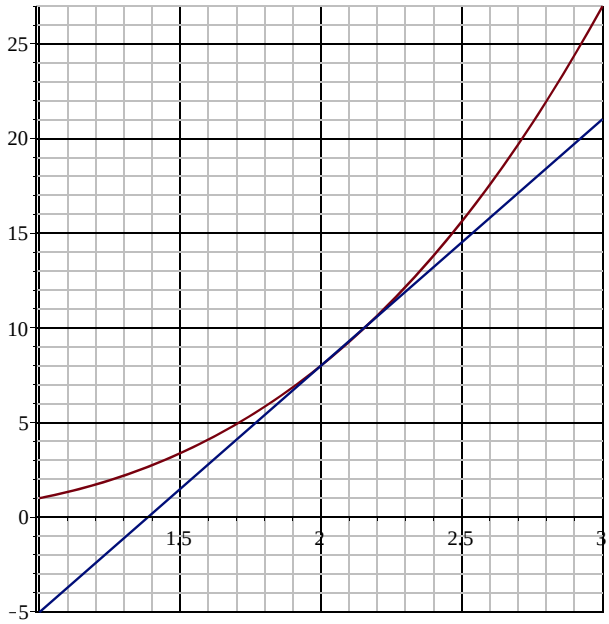
$$\frac{217}{16}$$



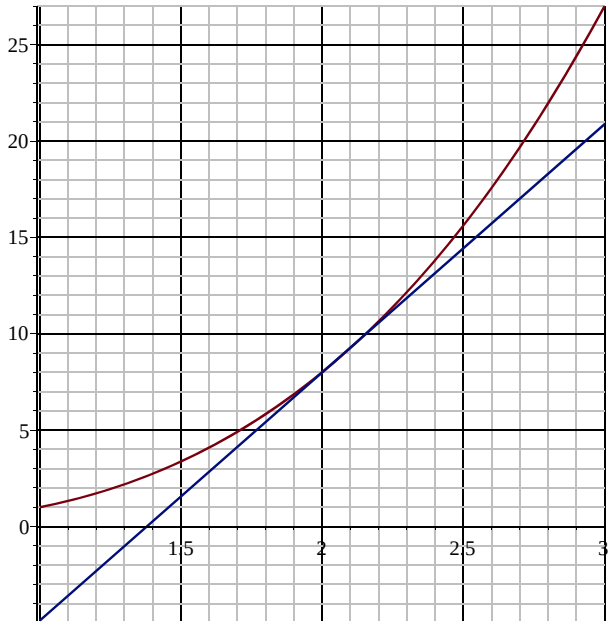
$$\frac{331}{25}$$



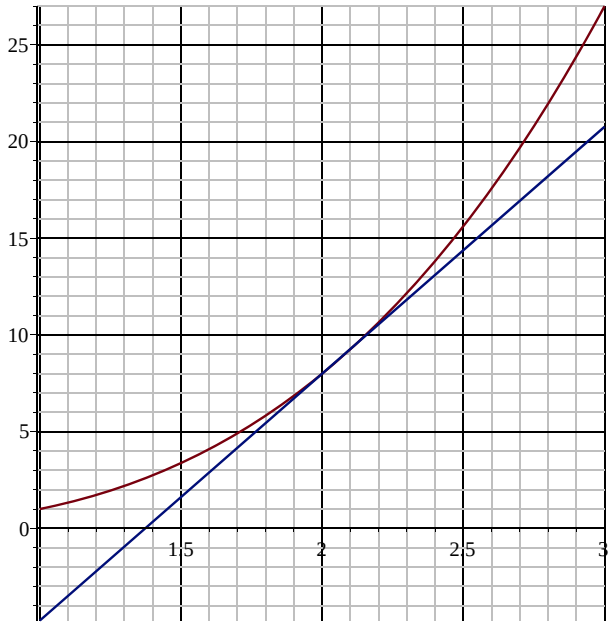
$$\frac{469}{36}$$



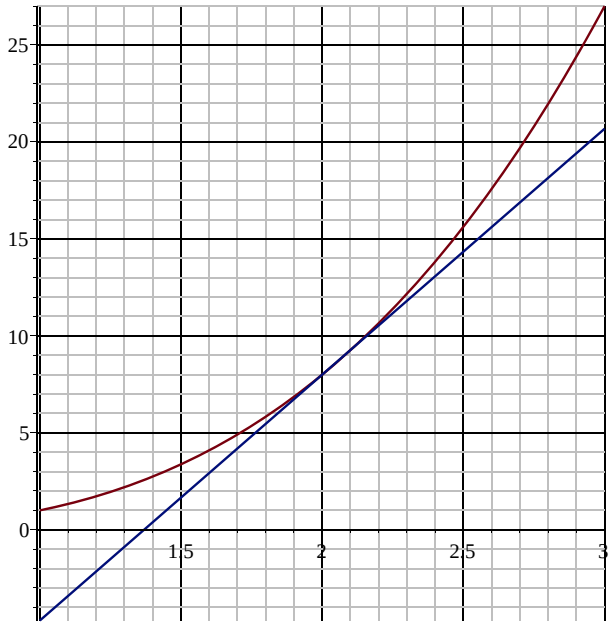
$$\frac{631}{49}$$



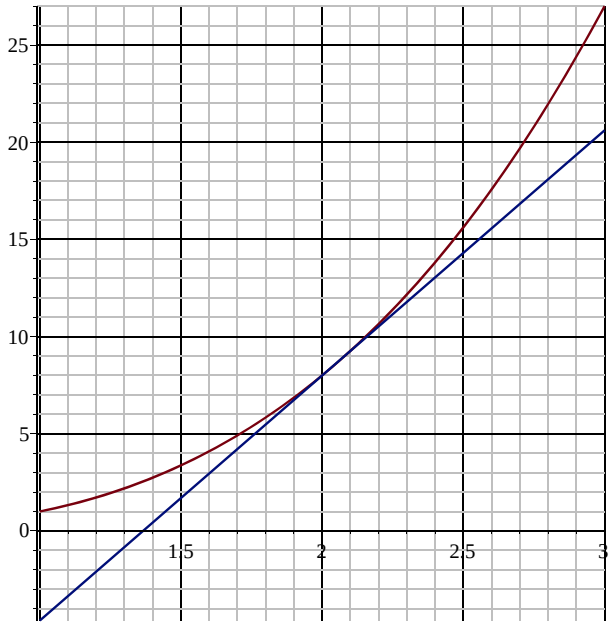
$$\frac{817}{64}$$



$$\frac{1027}{81}$$



$$\frac{1261}{100}$$



You should get a lot of pictures and definitions of slopes, m, above each picture. If you don't, it is probably because you forgot to define f.

Now scroll back up to see all the graphs you've made. You will see that these secant lines appear to repeat. In fact they don't quite repeat but get very close to a line called a tangent line. The last slope m is a good approximation of the slope of the tangent line.

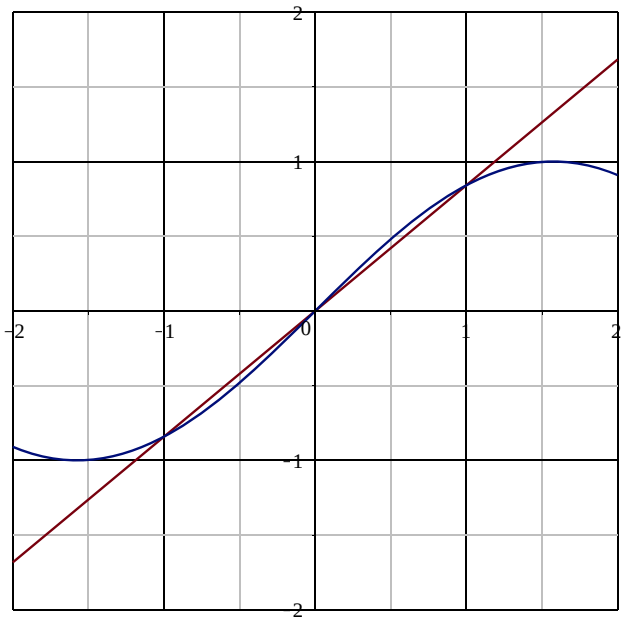
b) Now define a new function f(x) = sin(x) and use a loop like the one above to estimate the tangent line to sin(x) which passes through (0, sin(0)). Remember to look at secant lines which cross the graph of f above x=0 and above x=0+1/j and plot the curve over [-2,2] not [1,3]. ***Be very careful to save your work before hitting enter on a loop command.*** Then write a comment which gives the value of the slope of the tangent line and explains why.

```
> f:=x->sin(x);
```

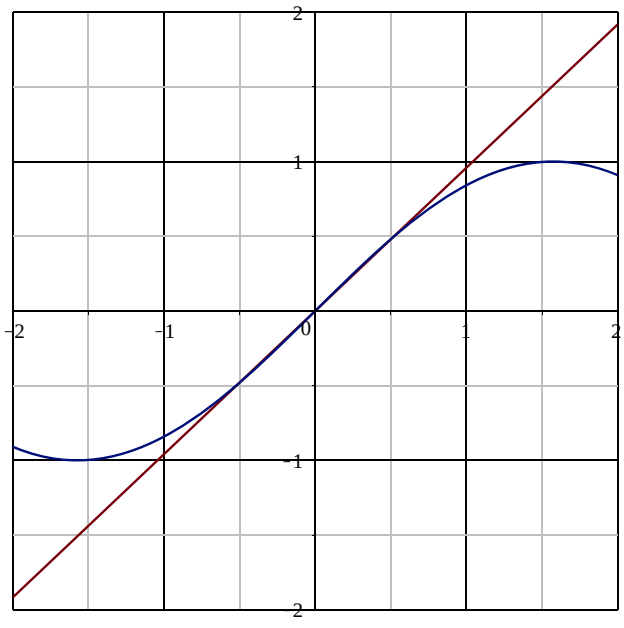
$$x \to \sin(x) \qquad \textbf{(8)}$$

```
> for j from 1 to 5 do m := (f(1/j)-f(0))/(1/j-0); plot({f(x), m*x}, x=-2..2, y=-2..2); od;
```

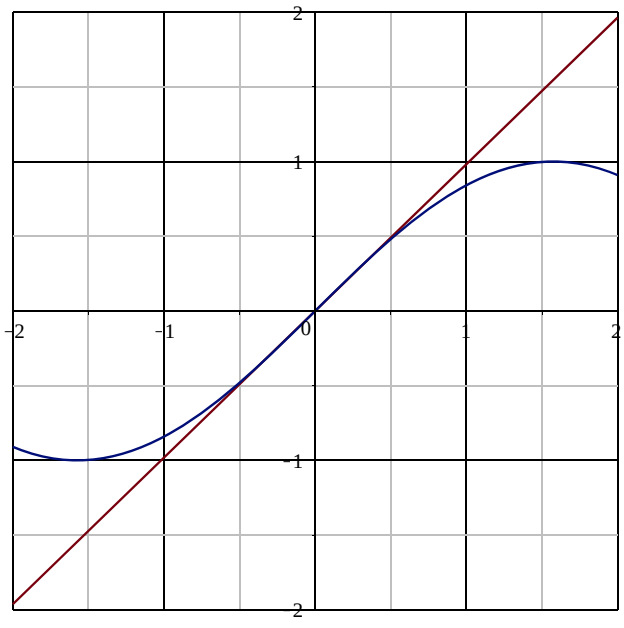
$$\sin(1)$$



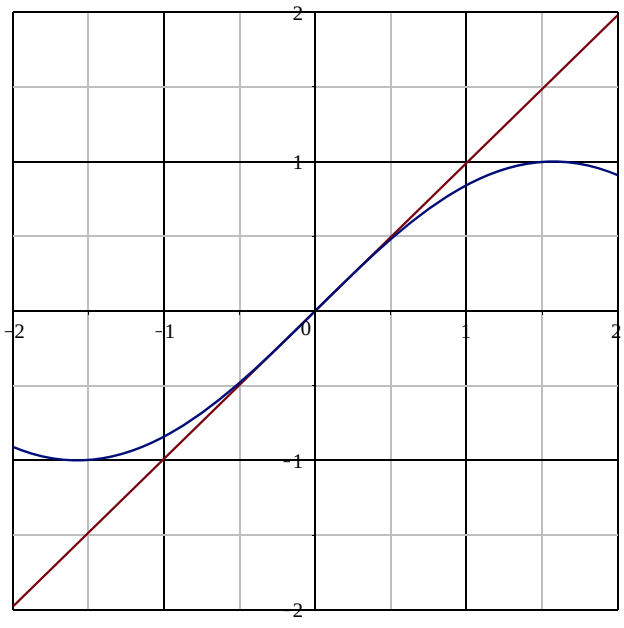
$$2 \sin\left(\frac{1}{2}\right)$$



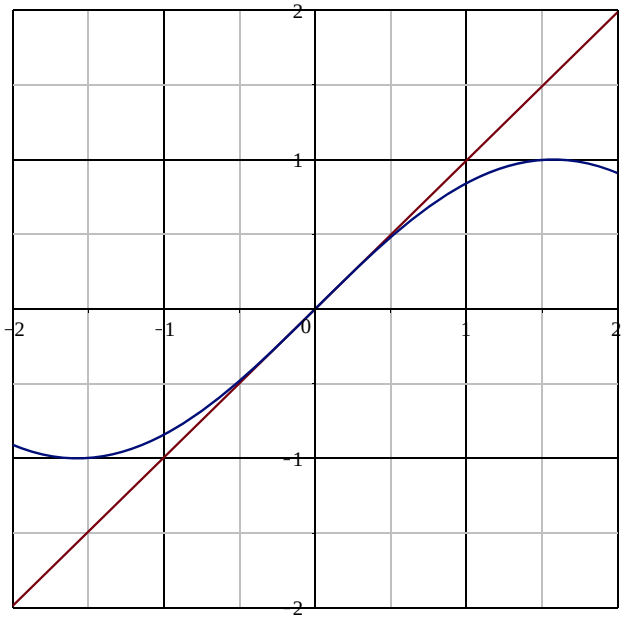
$$3 \sin\left(\frac{1}{3}\right)$$



$$4 \sin\left(\frac{1}{4}\right)$$



$$5 \sin\left(\frac{1}{5}\right)$$



Don't miss the colon after m!!

Exploration:

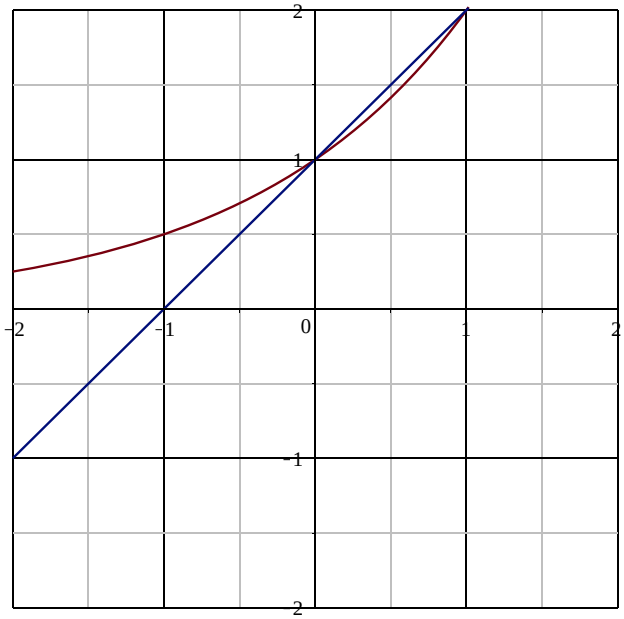
A: Tangent Lines to Exponential Functions:
Using the same loop that worked for f(x)=sin(x), find the slopes for 2^x, 3^x and e^x at x=0. Note that in maple the command exp(x) is the correct way to define e^x because maple does not understand the letter e. Write down your conclusions about the slopes and compare with ln(2) and ln(3) using the evalf command. ***Be very careful to save your work before hitting enter on a loop command.***

```
> f := x→2^x;
```

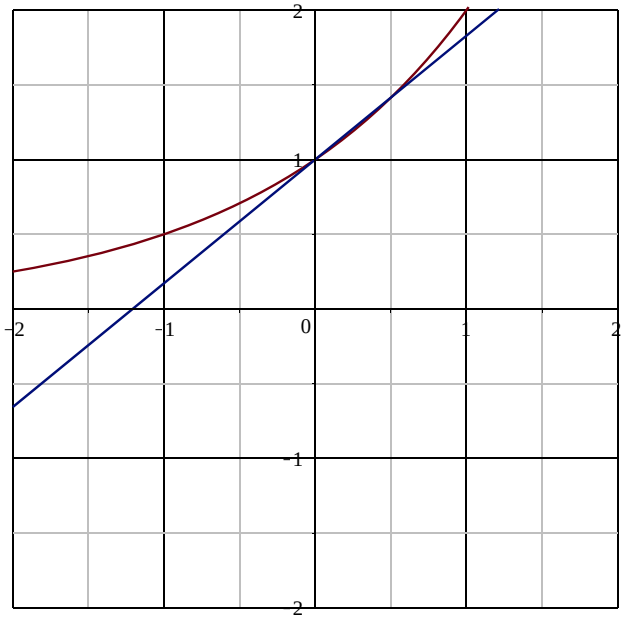
$$x \to 2^x \qquad \textbf{(9)}$$

```
> for j from 1 to 5 do m := evalf( (f(1/j)-f(0))/(1/j-0) ); plot( { f(x), m·x+1 }, x=-2..2, y=-2
  ..2 ); od;
```

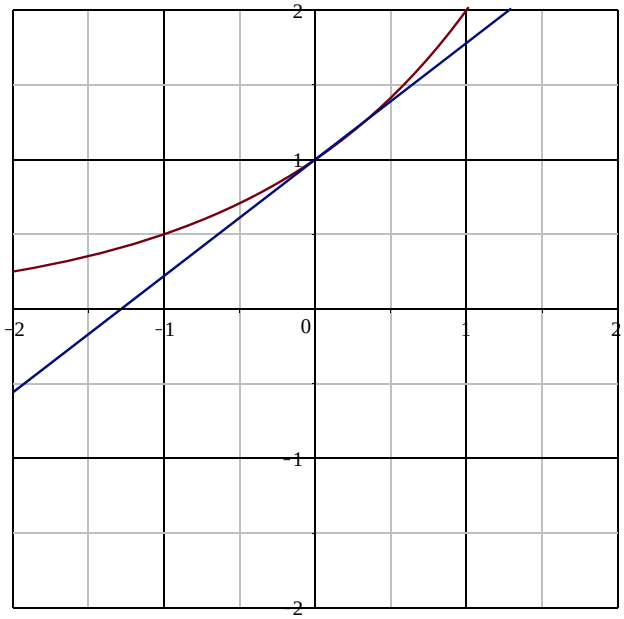
1.



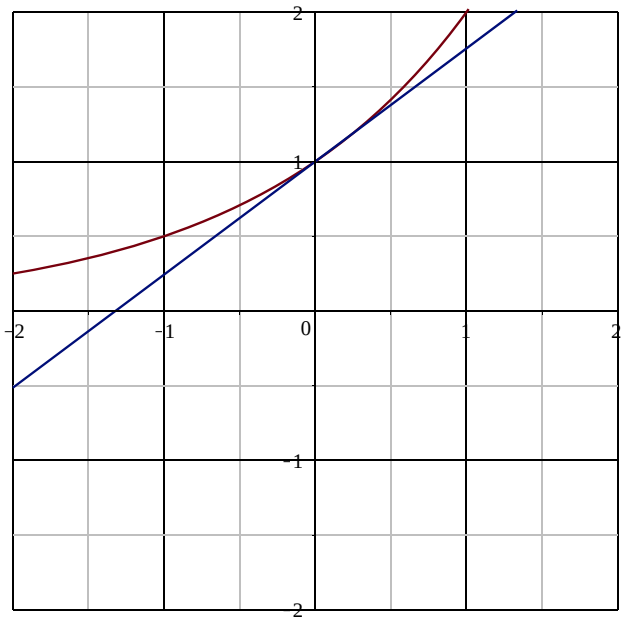
0.828427124



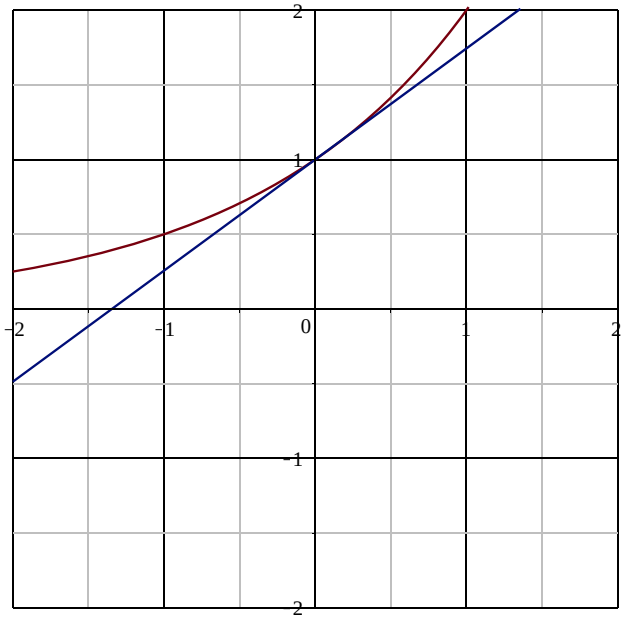
0.779763150



0.756828460



0.743491775




```
> evalf(ln(2));
```

0.6931471806 **(10)**

B: More Loops:
Use a loop to estimate the limit as x approaches 0 of g(x) and then test if your loop works by substituting various functions for g(x) and comparing your estimates with the ones found using maple's limit command > limit(g(x),x=a).

```
> f := x → (sin(x))/x;
```

$$x \to \frac{\sin(x)}{x} \quad \textbf{(11)}$$

```
> for k from 1 to 10 do evalf(f(10^(-k))) od;
```

0.9983341665 **(12)**
0.9999833334
0.9999998333
0.9999999983
1.000000000
1.000000000
1.000000000
1.000000000
1.000000000
1.000000000

```
> limit( f(x), x = 0);
```

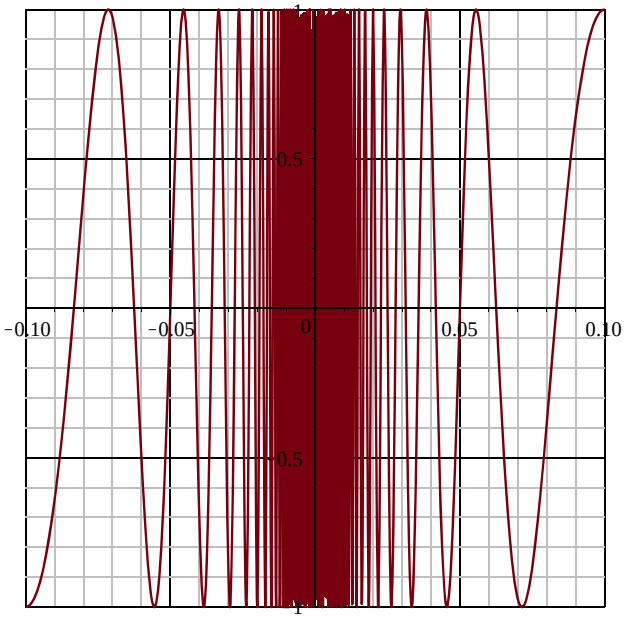
$$1 \qquad \textbf{(13)}$$

C: When the technique doesn't work: This method for finding tangent lines by taking secant lines through (a, f(a)) and (a+1/j, f(a+1/j)) does not work if the tangent line doesn't exist at a . **If the tangent line to f(x) at a does not exist then two things might happen: 1)** The various plots of the secant lines will not look like each other even if you take j very large (try a=0, f(x)=sin(Pi/4x)) or **2)** the secant lines will get closer and closer to a line but it won't be a tangent line (try a=0 f(x)=abs(x)). ***Be very careful to save your work before hitting enter on a loop command.***

```
> f := x→sin(π/(4 x));
```

$$x \to \sin\left(\frac{1}{4}\,\frac{\pi}{x}\right) \qquad \textbf{(14)}$$

```
> plot( f(x), x =-.1 ...1);
```

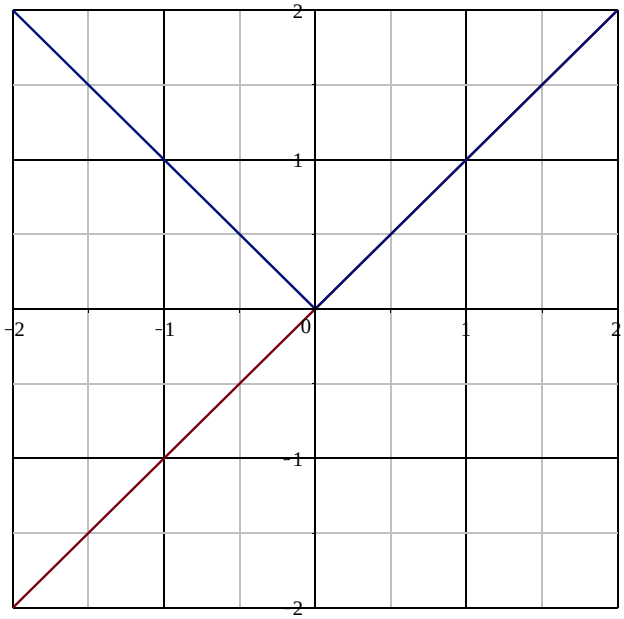



```
> for j from 1 to 2 do m := evalf( ( f(1/j) - f(0) ) / ( 1/j - 0 ) ); plot( { f(x), m·x + f(0) }, x =-2 ..2, y =-2
..2); od;
Error, (in f) numeric exception: division by zero
> f := x→abs(x);
```

$$x \to |x| \qquad \textbf{(15)}$$

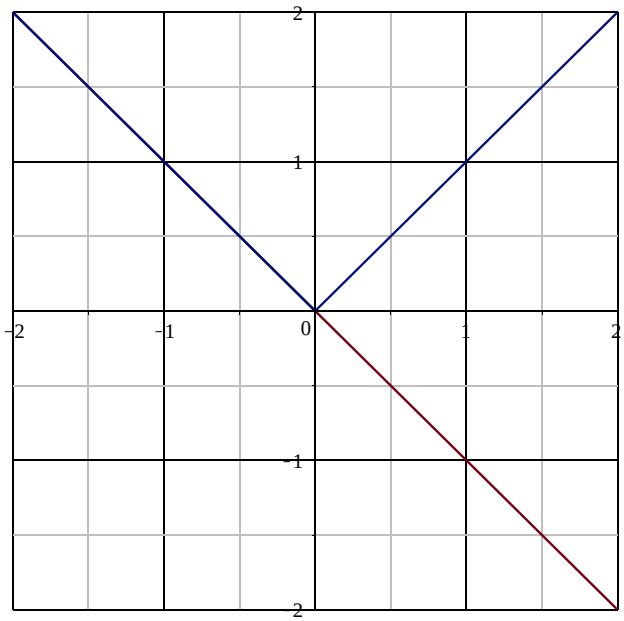
> **for** j **from** 1 **to** 1 **do** $m := evalf\left(\frac{\left(f\left(\frac{1}{j}\right)-f(0)\right)}{\left(\frac{1}{j}-0\right)}\right)$; $plot(\{f(x), m\cdot x+f(0)\}, x=-2..2, y=-2..2)$; **od**;

1.



> **for** j **from** −1 **to** −1 **do** $m := evalf\left(\frac{\left(f\left(\frac{1}{j}\right)-f(0)\right)}{\left(\frac{1}{j}-0\right)}\right)$; $plot(\{f(x), m\cdot x+f(0)\}, x=-2..2, y=-2..2)$; **od**;

−1.


2
1
-2
-1
0
1
2
-1
-2

> ?