MAT 155 Project IX Spring 2014

Increasing and Decreasing

BYUNG DO PARK

Your work should be done neatly in the same format as previous labs and saved regularly as yourname9.mws and as yourname9b.mws.

You should write comments as you work. To do so just backspace in front of the prompt > and type in the comments. They should appear in black not in red. When you finish the lab be sure to go over your comments and check the grammar.

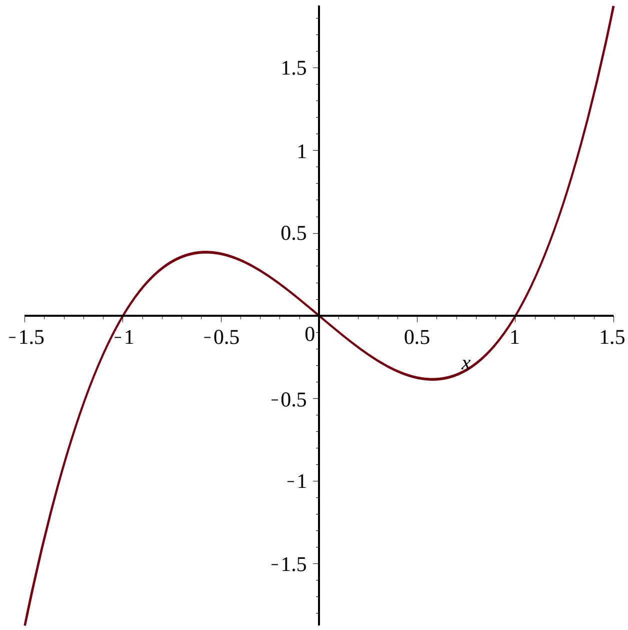
Problem 1: Local Maxima and Critical Points

a) Define a function f which takes x to x^3-x.
b) Graph it from -1.5 to 1.5. Write a comment explaining where the function increases and where it decreases. Try to estimate where it switches from increasing to decreasing. Such points are called local maxima. What is the slope at the points where it switches? Now check where it switches from decreasing to increasing. These are the local minima. What is the slope of f at the local minima?
c) Find its derivative using the D command and graph the derivative from -1.5 to 1.5 using
> plot(D(f)(x), x=-1.5..1.5);
d) Write a comment discussing where the derivative is positive and where it is negative. Use solve to find out where the derivative is 0. A points where the derivative is zero is called a critical point. How are the critical points you've found related to the graph of f ? How are they related to the local minima and maxima?

```
> f:=x->x^3-x;
```

$$f := x \rightarrow x^3 - x \quad (1)$$

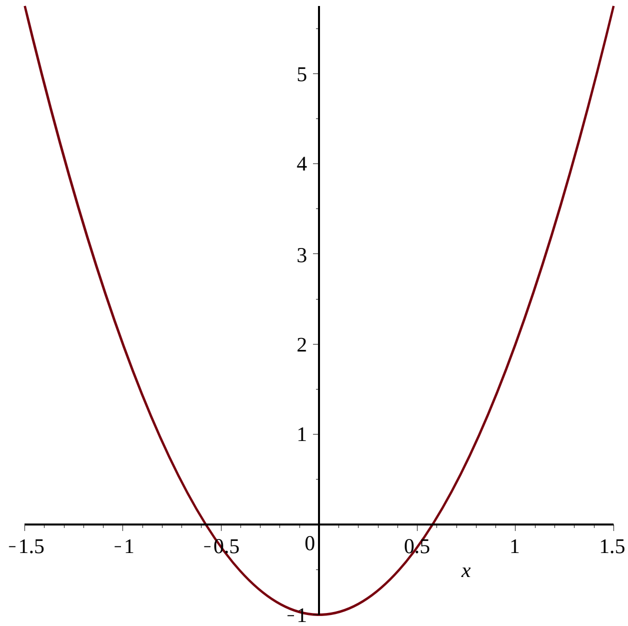
```
> plot(f(x),x=-1.5..1.5);
```




```
> D(f);
```

$$x \rightarrow 3\,x^2 - 1 \qquad (2)$$

```
> plot(D(f)(x),x=-1.5..1.5);
```




```
> criticalnum:=solve(D(f)(x)=0,x);
```

$$criticalnum := \frac{1}{3}\sqrt{3}, -\frac{1}{3}\sqrt{3} \quad (3)$$

```
> f(criticalnum[1]); f(criticalnum[2]);
```

$$-\frac{2}{9}\sqrt{3}$$

$$\frac{2}{9}\sqrt{3} \quad (4)$$

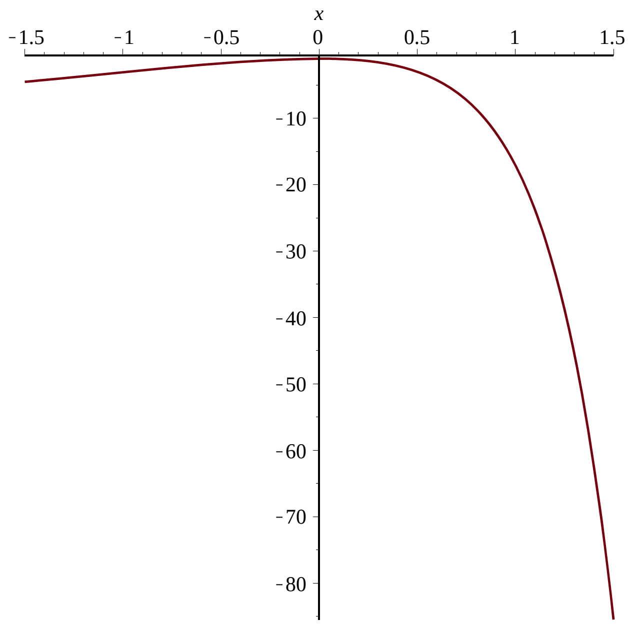
```
> restart;
```

Problem 2: *Don't forget to restart.*
Repeat problem 1 with f(x)= 3x-exp(3x). You may need to use fsolve in step d.

```
> f:=x->3*x-exp(3*x);
```

$$f := x \rightarrow 3\,x - e^{3\,x} \quad (5)$$

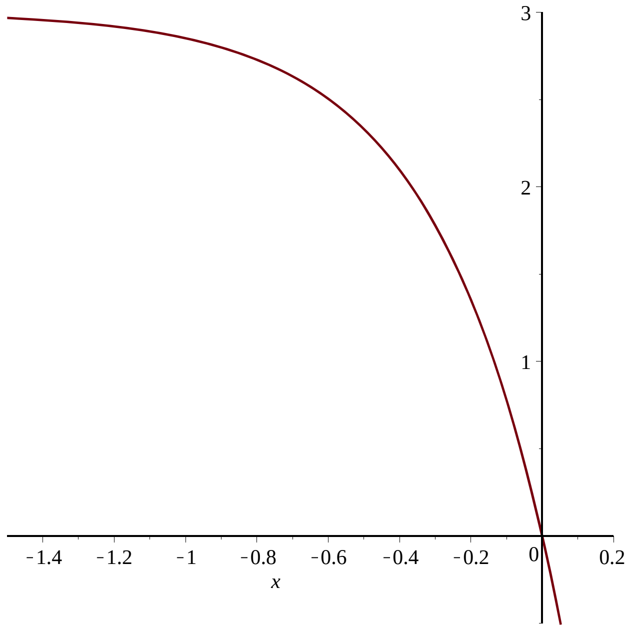
```
> plot(f(x),x=-1.5..1.5);
```




```
> D(f);
```

$$x \rightarrow 3 - 3\,\mathrm{e}^{3\,x} \qquad (6)$$

```
> plot(D(f)(x),x=-1.5..1.5);
```




```
> fsolve(D(f)(x)=0,x);
```

$$0. \quad (7)$$

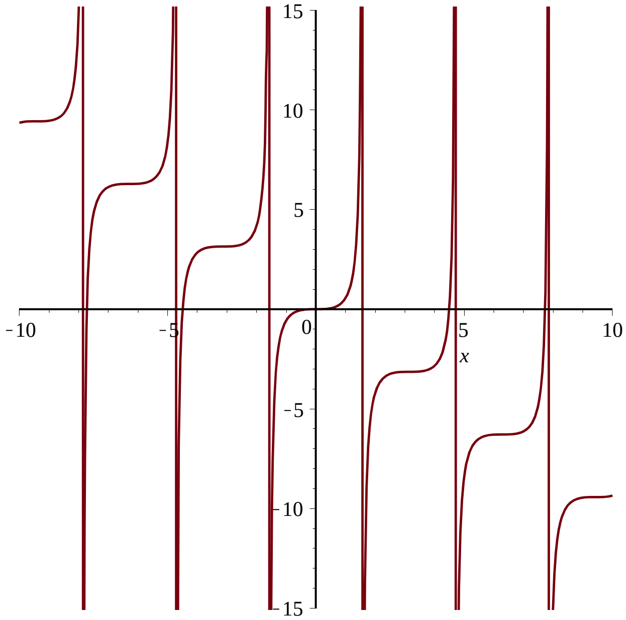
```
> restart;
```

Problem 3: A special critical point:
Repeat problem 1 with f(x)=tan(x)-x. Notice that this function is always increasing but the slope is zero at one point between -1.5 and 1.5. Explain. Can a point be a critical point even though it isn't a local minimum nor a local maximum? Examine the graph near that point. What is its linear approximation at that point?

```
> f:=x->tan(x)-x;
```

$$f := x \rightarrow \tan(x) - x \quad (8)$$

```
> plot(f(x),x=-10..10);
```



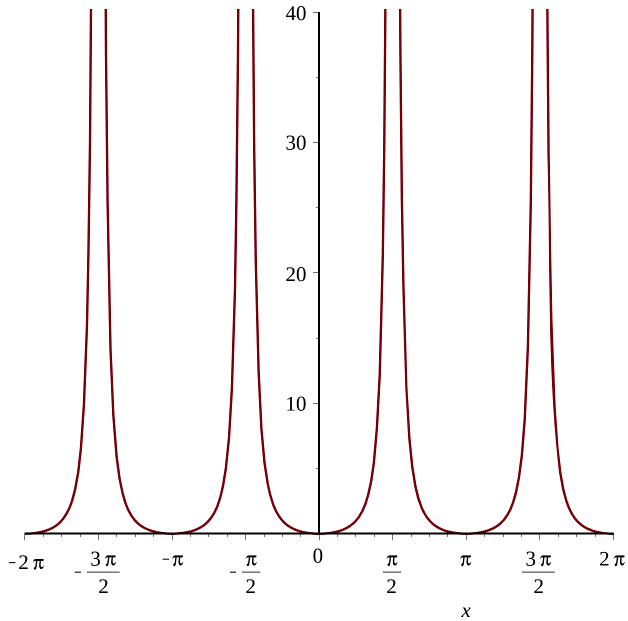

```
> D(f);
```

$$x \rightarrow \tan(x)^2 \quad \textbf{(9)}$$

```
> fsolve(D(f)(x)=0,x=-1..2);
```

$$0. \quad \textbf{(10)}$$

```
> plot(D(f)(x));
```



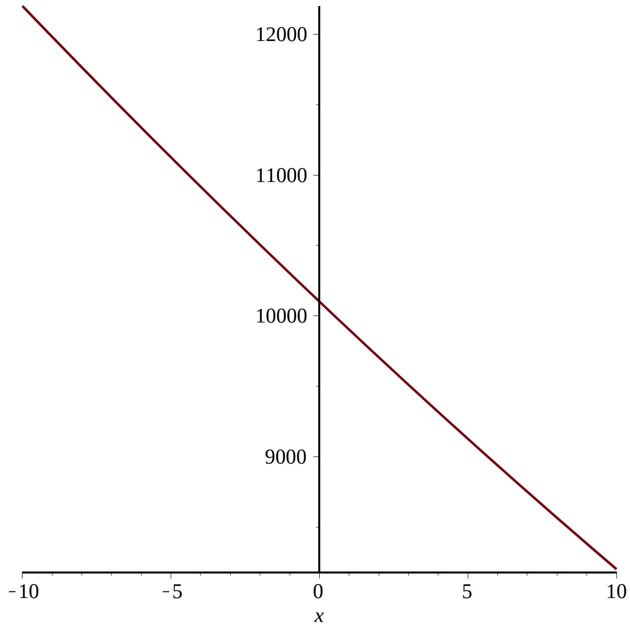
Problem 4: Locating the interesting part of a graph

a) The graph of f(x)=x^2-200x+10101 is a parabola but it is difficult to choose good x and y bounds to make it look like one. The best way to graph it well is to find the location of its local minimum. A graph switches from decreasing to increasing at a local minimum, so it has a critical point there. You can find it by solving D(f)(x)=0. Then graph f, making sure to include the local minimum.

b) repeat part a) with f(x)= 202x-x^2 and comment.

```
> f:=x->x^2-200*x+10101; plot(f(x));
```

$$f := x \rightarrow x^2 - 200\,x + 10101$$




```
> D(f);
```

$$x \to 2\,x - 200 \quad (11)$$

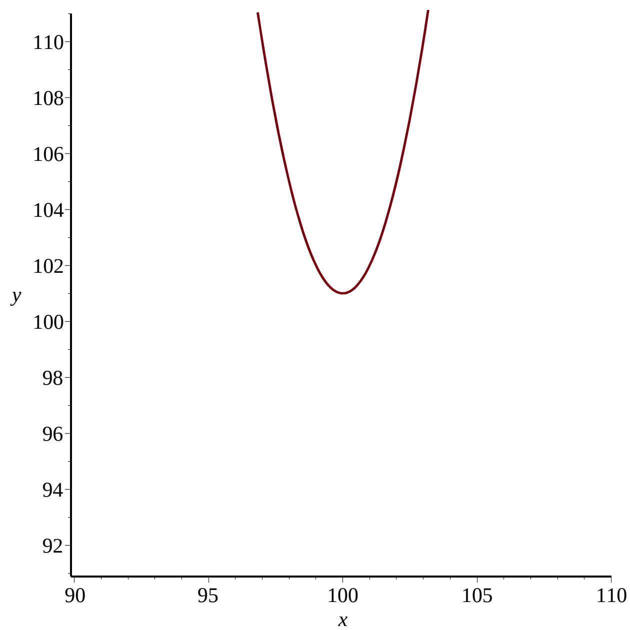
```
> solve(D(f)(x));
```

$$100 \quad (12)$$

```
> f(100);
```

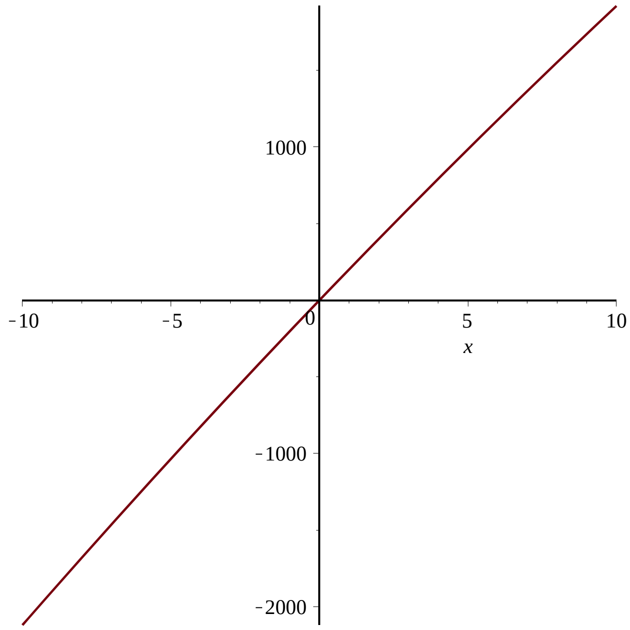
$$101 \quad (13)$$

```
> plot(f(x), x=90..110, y=91..111);
```




```
> f:=x-> 202*x-x^2; plot(f(x));
```

$$f := x \rightarrow 202\,x - x^2$$




```
> D(f);
```

$$x \rightarrow -2\,x + 202 \tag{14}$$

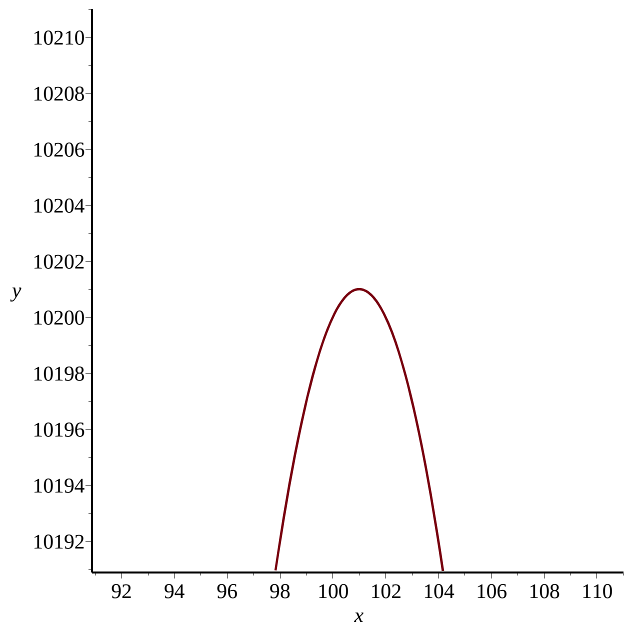
```
> solve(D(f)(x),x);
```

$$101 \tag{15}$$

```
> f(101);
```

$$10201 \tag{16}$$

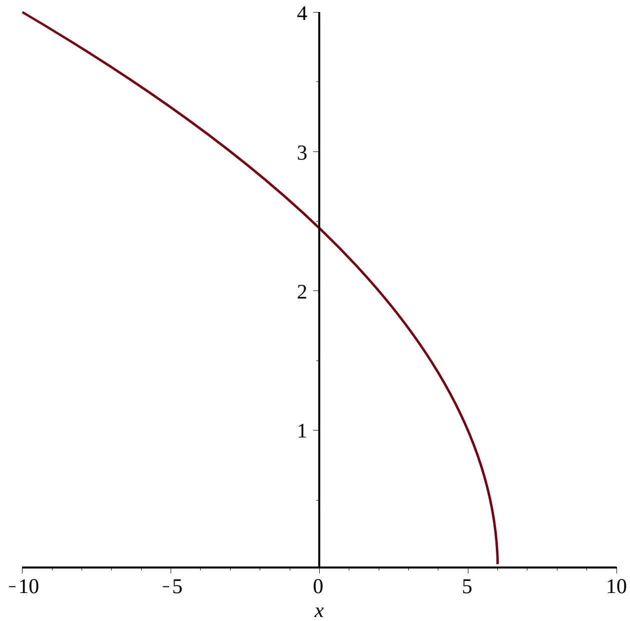
```
> plot(f(x),x=91..111, y=10191..10211);
```



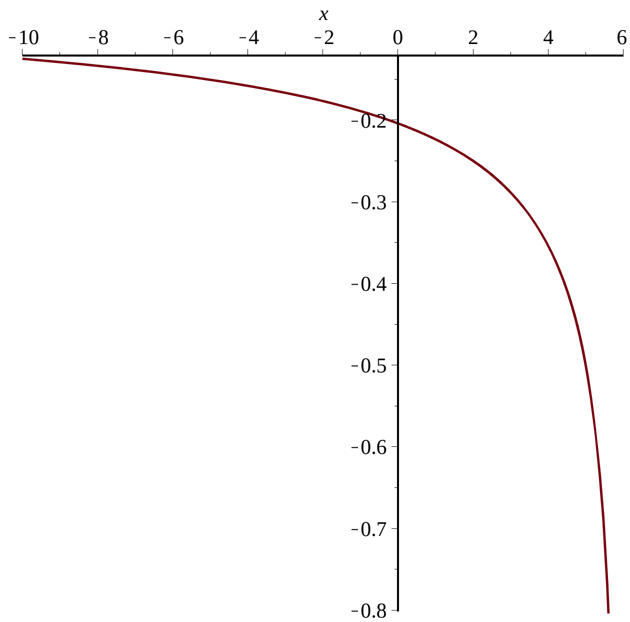
Exploration: Examine other graphs of functions and their derivatives to study where the function increases and decreases. Some functions to look at are sqrt(6-x), 1/x, x-2/x and/or 1/(1+x^2).

```
> f:=x->sqrt(6-x); plot(f(x)); D(f); plot(D(f)(x));
```

$$f := x \rightarrow \sqrt{6 - x}$$

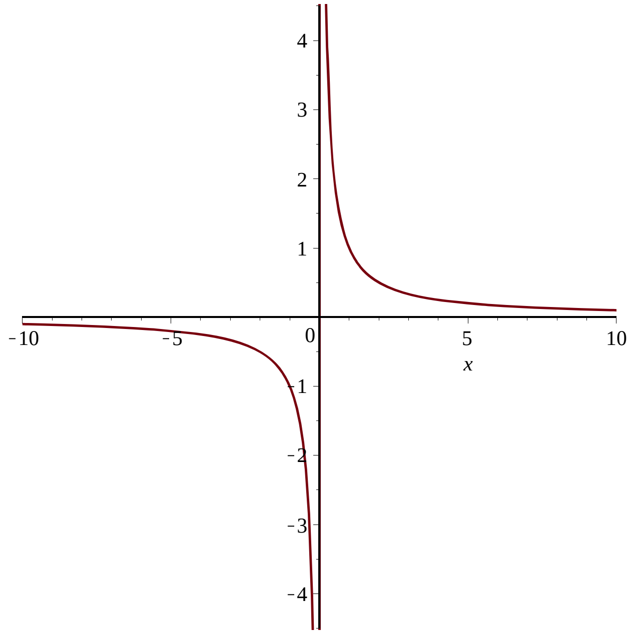


$$x \rightarrow -\frac{1}{2\sqrt{6-x}}$$

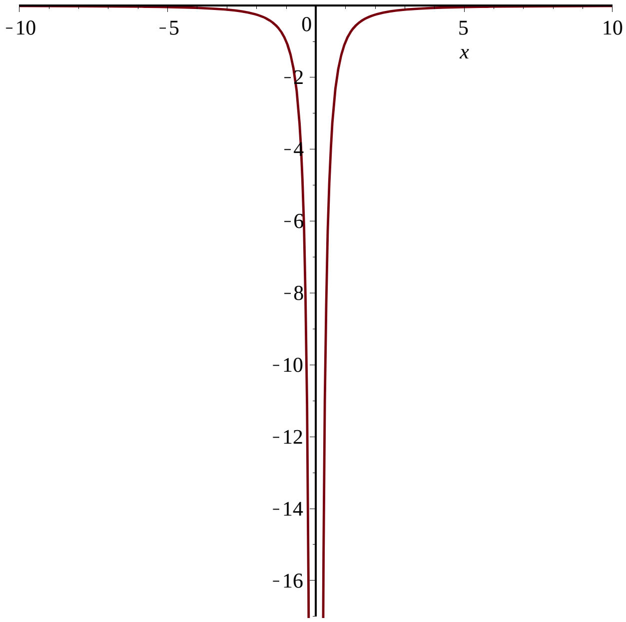



```
> f:=x->1/x; plot(f(x)); D(f); plot(D(f)(x));
```

$$f := x \rightarrow \frac{1}{x}$$

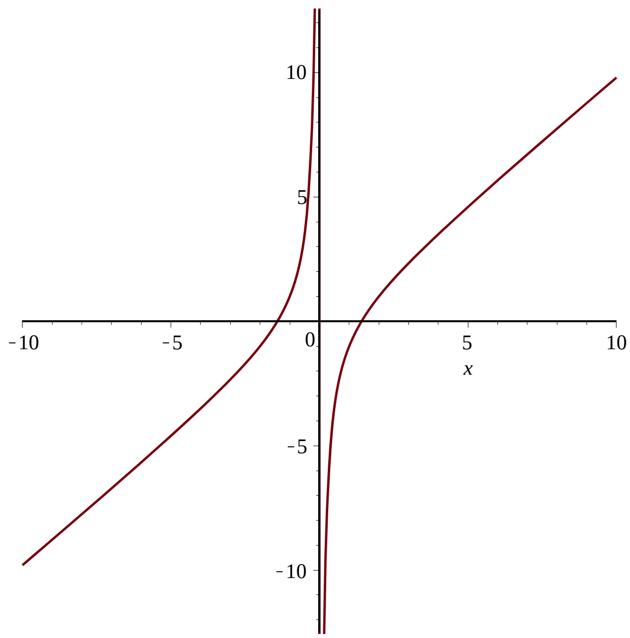


$$x \rightarrow -\frac{1}{x^2}$$

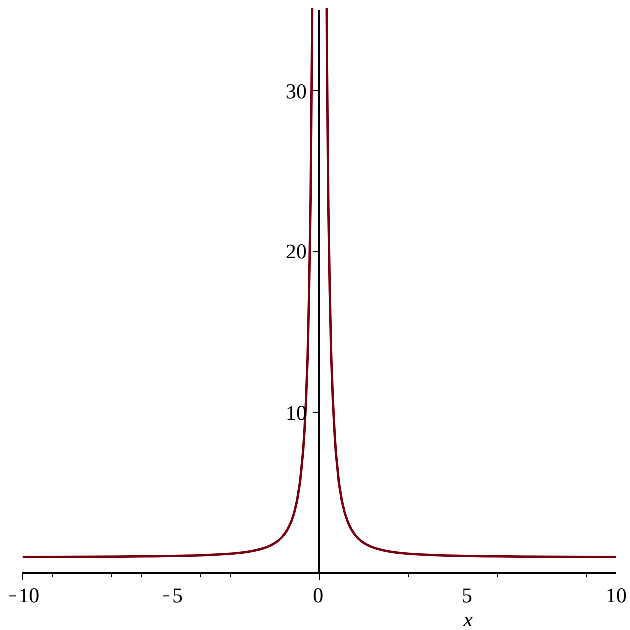



```
> f:=x->x-2/x; plot(f(x)); D(f); plot(D(f)(x));
```

$$f := x \rightarrow x - \frac{2}{x}$$

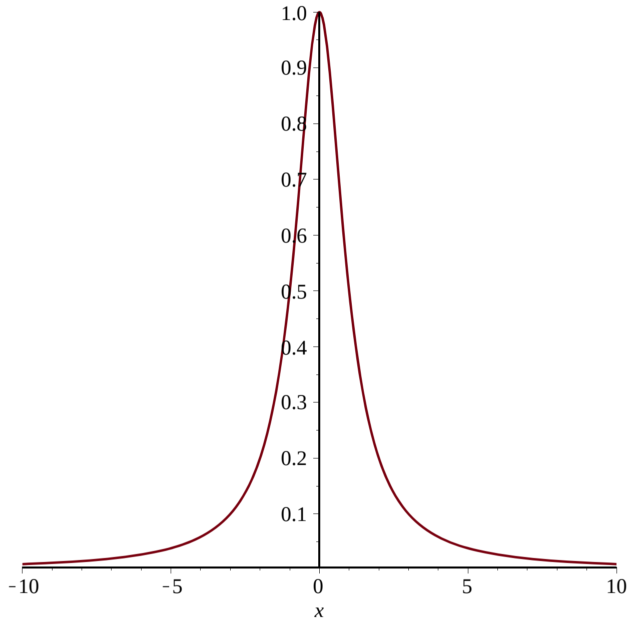


$$x \rightarrow 1 + \frac{2}{x^2}$$

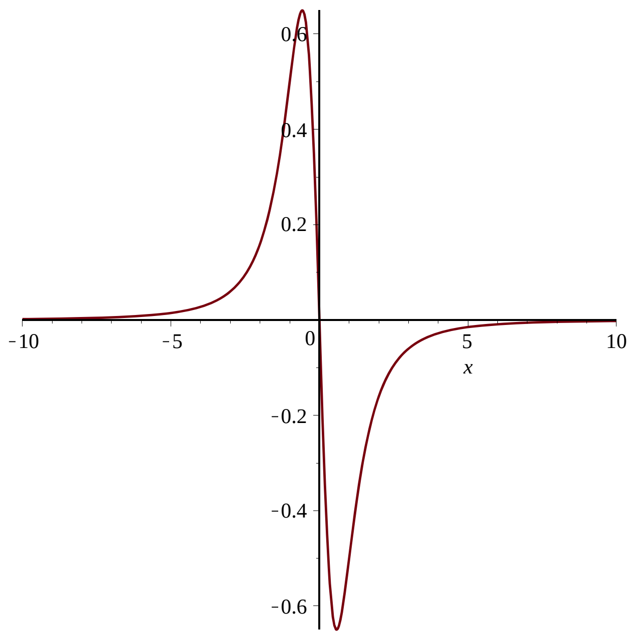



```
> f:=x->1/(1+x^2); plot(f(x)); D(f); plot(D(f)(x));
```

$$f := x \rightarrow \frac{1}{1+x^2}$$



$$x \rightarrow -\frac{2x}{\left(1+x^2\right)^2}$$



0.6
0.4
0.2
-10
-5
0
5
10
x
-0.2
-0.4
-0.6