Midterm Exam I
Spring 2014, MAT155 Section B401[19441]
March 5th, 2014. 11:00AM--12:40PM.

Instructions: You can use only MAPLE program and a web browser(only for the purpose of submitting your exam solution to the instructor). You may not use any other programs other than these two. You can look up your MAPLE source files, but otherwise this exam is closed-book, closed-note, and you may not use any electronic device in this exam except your PC. You are not allowed to talk to other students. Type all details explicitly. All solutions should be obtained by using MAPLE codes.

Problem 1. Define a function f(x)=x^3 and calculate the following.
(1) (sin 60 degree)^3 and (tan(Pi/4))^3 (5 points)

```
> f:=x->x^3;
```

$$x \rightarrow x^3 \tag{1}$$

```
> f(sin(Pi/3)); f(tan(Pi/4));
```

$$\frac{3}{8}\sqrt{3} \tag{2}$$
$$1$$

(2) g(f(x)) and f(g(x)) when g(x) is defined by the cubic root of x. (5 points)

```
> g:=x->x^(1/3);
```

$$x \rightarrow x^{1/3} \tag{3}$$

```
> g(f(x)); f(g(x));
```

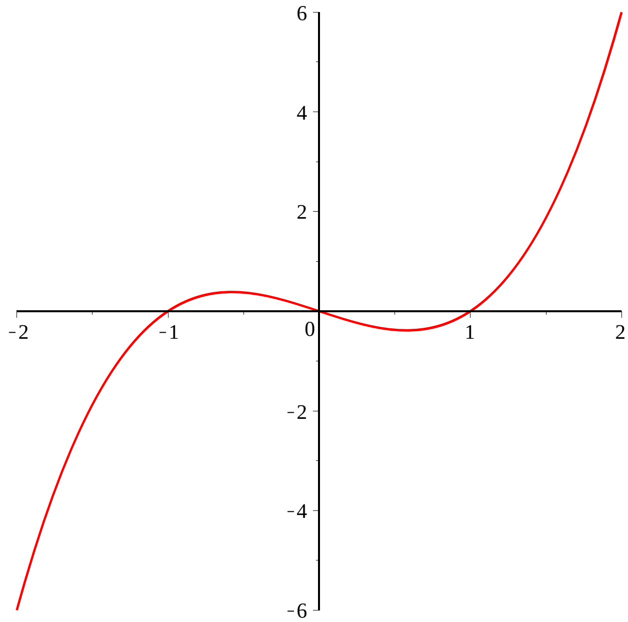
$$\left(x^3\right)^{1/3} \tag{4}$$
$$x$$

Problem 2. Plot the following functions so that your graph will include all zeros of the function:
(1) f(x)=x^3- x (5 points)

```
> f:=x^3-x;
```

$$x^3 - x \tag{5}$$

```
> plot(f(x),x=-2..2);
```

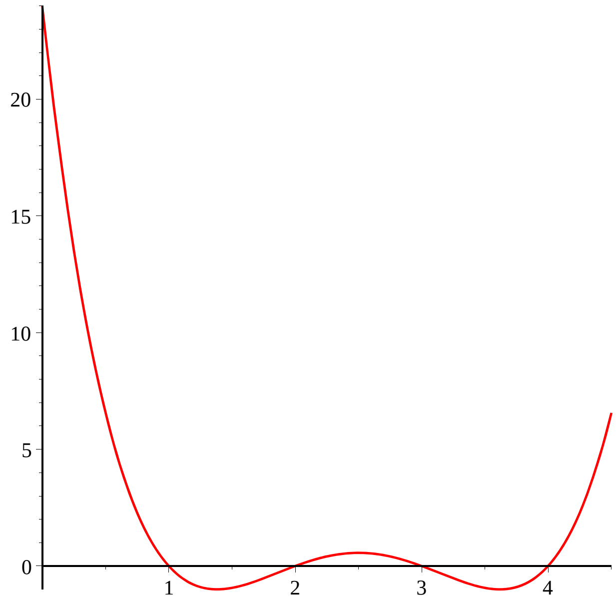


(2) g(x)=(x-1)(x-2)(x-3)(x-4) (5 points)

```
> g:=x->(x-1)*(x-2)*(x-3)*(x-4);
```

$$x \rightarrow (-1 + x)\ (x - 2)\ (x - 3)\ (x - 4) \tag{6}$$

```
> plot(g(x), x=0..4.5);
```

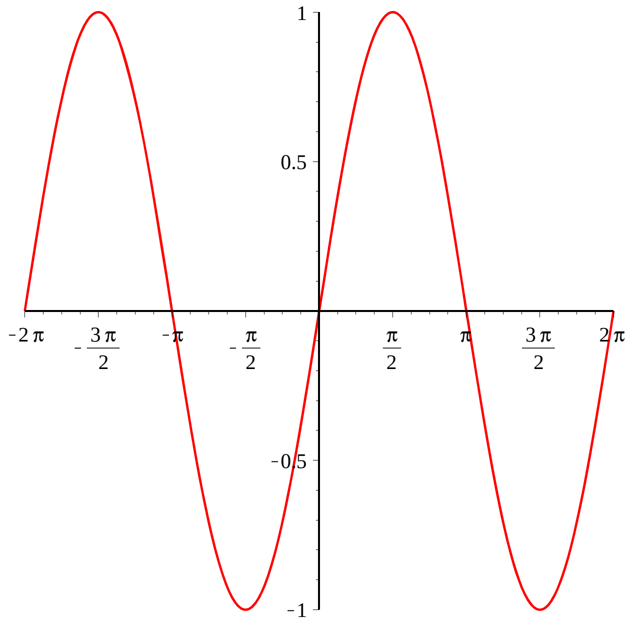


Problem 3. Plot the following functions for the given values:
(1) f(x)=sin(x) for x is in [-2*Pi,2*Pi] (5 points)

```
> f:=x->sin(x);
```

$$x \rightarrow \sin(x) \quad \textbf{(7)}$$

```
> plot(f(x),x=-2*Pi..2*Pi);
```

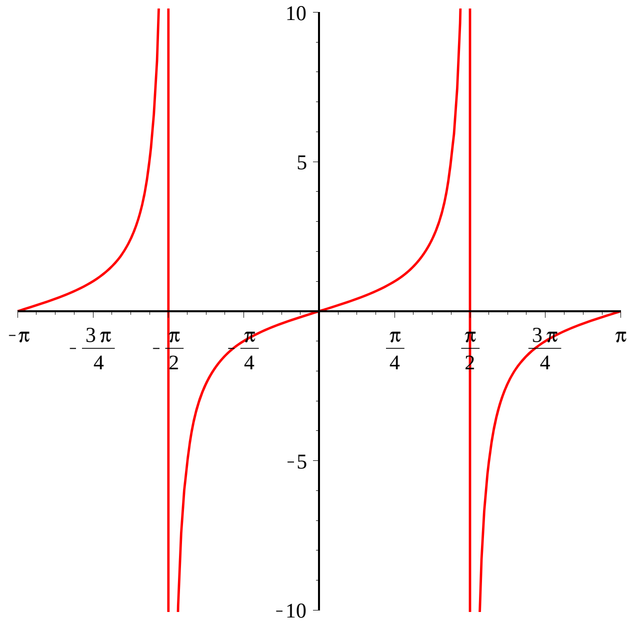


(2) g(x)=tan(x) for x is in [-Pi,Pi] (5 points)

```
> g:=x->tan(x);
```

$$x \rightarrow \tan(x) \tag{8}$$

```
> plot(g(x),x=-Pi..Pi, y=-10..10);
```

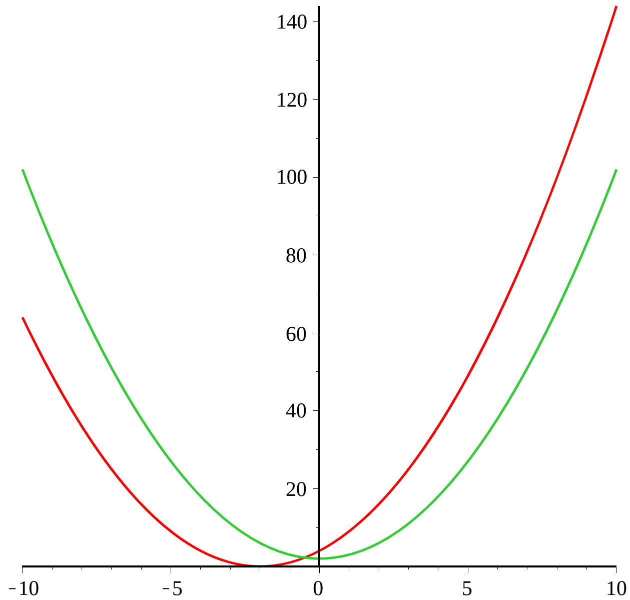


Problem 4. Let f(x)=x^2. Then graph f(x+2), and f(x)+2 and compare them. (10 points; 5 points each)

```
> f:=x->x^2;
```

$$x \rightarrow x^2 \tag{9}$$

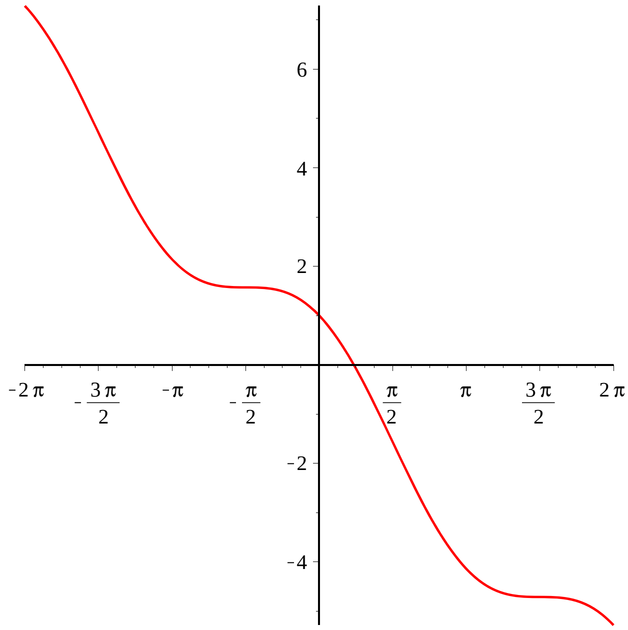
```
> plot({f(x+2),f(x)+2});
```



Problem 5. Define a function f(x)=cos(x)-x, plot f(x), and solve f(x)=0. (Hint: Use fsolve command.) (10 points)

```
> f:=x->cos(x)-x; plot(f(x)); fsolve(f(x)=0,x);
```

$$x \rightarrow \cos(x) - x$$

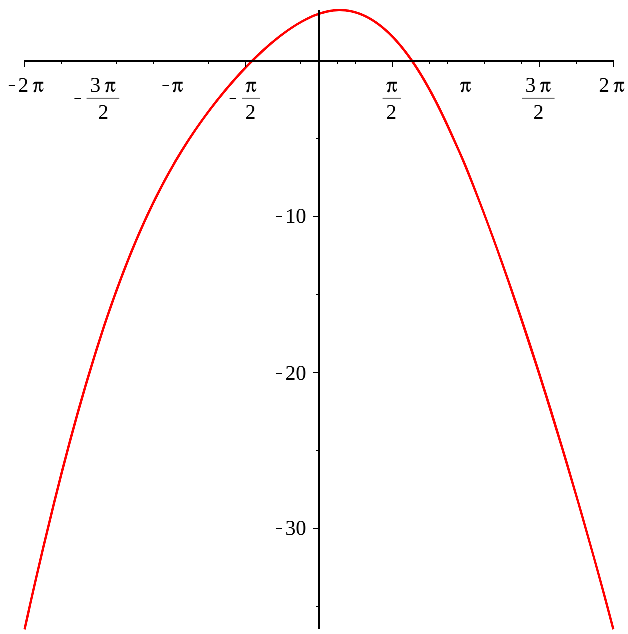


$$0.7390851332$$

Problem 6. Let f(x)= sin(x)-x^2+3 and solve for f(x) less than or equal to zero. (10 points)

```
> f:=x->sin(x)-x^2+3; plot(f(x)); fsolve(f(x)=0,x=-4..0); fsolve(f
  (x)=0,x=0..4);
```

$$x \rightarrow \sin(x) - x^2 + 3$$



$$-1.418310092$$
$$1.979320147$$

Hence f(x) is less than or equal to zero on intervals (-infinity,-1.418310092] and [

Problem 7. Find the inverse function of f(x)=(x+2)/x. Use MAPLE to graph both f and f^{-1} in the same viewing window. (10 points)

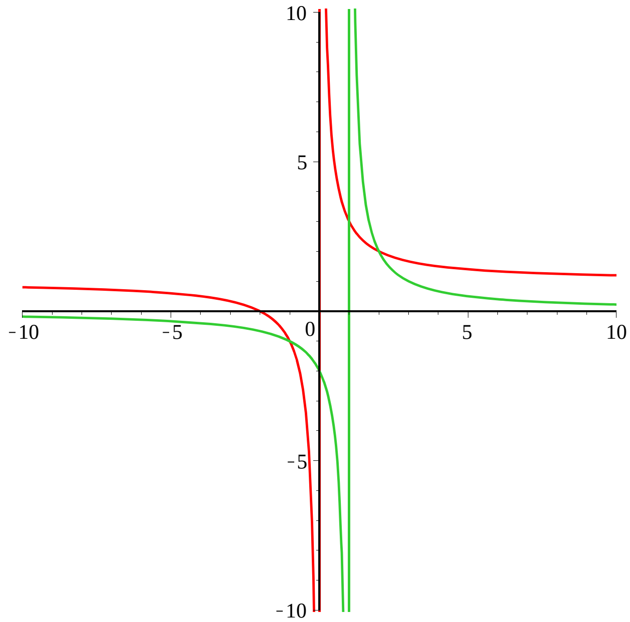
```
> f:=x->(x+2)/x; solve(f(y)=x,y); invf:=x->2/(-1+x);
```

$$x \rightarrow \frac{x+2}{x} \tag{10}$$

$$\frac{2}{-1+x}$$

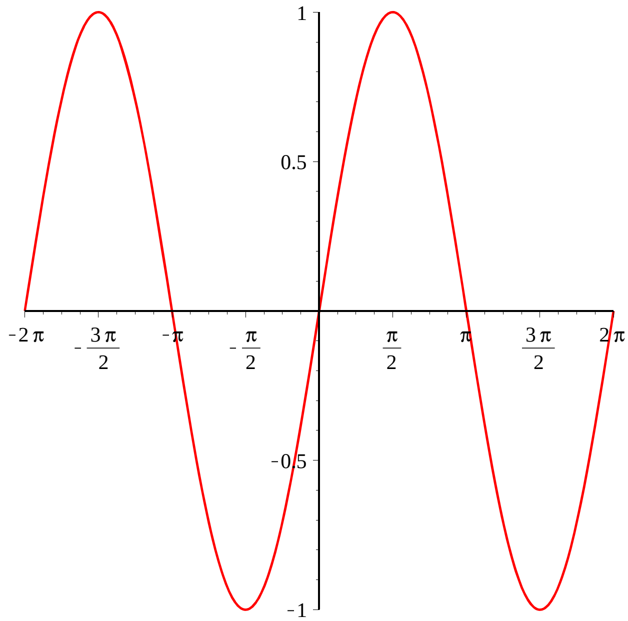
$$x \rightarrow \frac{2}{-1+x}$$

```
> plot({f(x),invf(x)}, x=-10..10, y=-10..10);
```

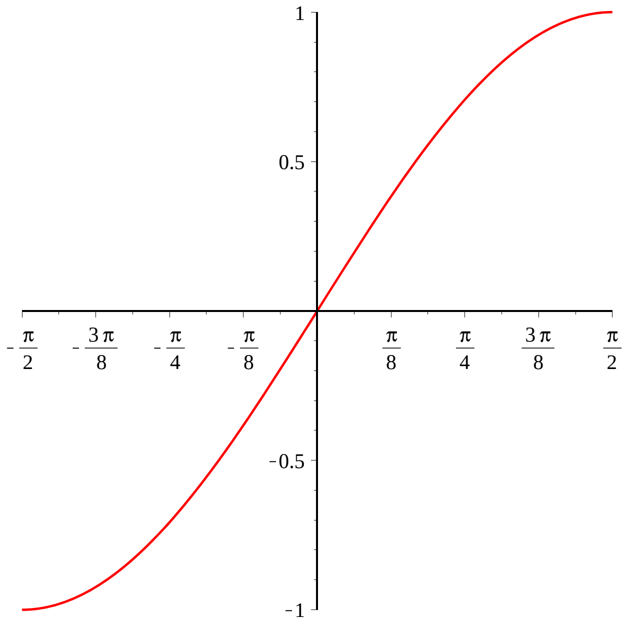


Problem 8. (1) Graph sin(x) on [-10,10]. Choose a restricted domain for sin(x) that includes x=0 and passes the horizontal line test. (5 Points)

```
> plot(sin(x));
```

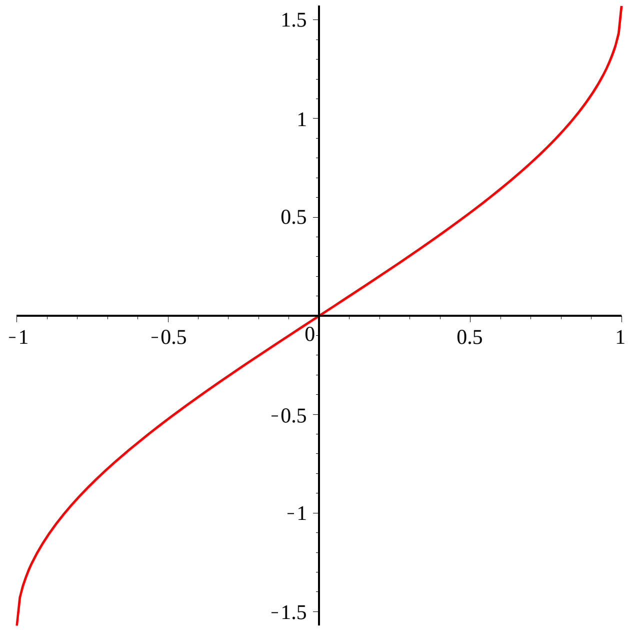



```
> plot(sin(x),x=-Pi/2..Pi/2);
```



(2) Draw the graph of arcsin(x) and discuss the relationship with the graph obtained in (1) above. (5 Points)

```
> plot(arcsin(x),x=-1..1);
```



We can obtain the graph of (2) by reflecting the graph of (1) to the straightline y=x.

Problem 9. (1) For f(x)=(x^4+14*x^3+71*x^2+154*x+120)/(x^3+6*x^2+11*x+6), f(x) has a vertical asymptote x=k. Find the constant k.(5 points)
(2) Evaluate limit of f(x) as x goes to k where k is the constant obtained in (1) (5 points)

```
> f:=x->(x^4+14*x^3+71*x^2+154*x+120)/(x^3+6*x^2+11*x+6);
```

$$x \rightarrow \frac{x^4+14\,x^3+71\,x^2+154\,x+120}{x^3+6\,x^2+11\,x+6} \qquad \textbf{(11)}$$

```
> simplify(f(x));
```

$$\frac{x^2+9\,x+20}{x+1} \qquad \textbf{(12)}$$

The vertical asymptote: x=-1. Hence k=-1.

Problem 10. (1) Evaluate limits of f(x)=(1-tan(x))/(sin(x)-cos(x)) as x goes to Pi/4. (5 Points)
(2) Evaluate limits of f(x)=(1-exp(-x))/(exp(x)-1) as x goes to 0. (5 Points)

```
> f:=x->(1-tan(x))/(sin(x)-cos(x));limit(f(x),x=Pi/4);
```

(13)

$$x \rightarrow \frac{1-\tan(x)}{\sin(x)-\cos(x)} \tag{13}$$

$$-\sqrt{2}$$

```
> f:=x->(1-exp(-x))/(exp(x)-1); limit(f(x),x=0);
```

$$x \rightarrow \frac{1-e^{-x}}{e^{x}-1} \tag{14}$$

$$1$$

> ?