

# MAT 156 LAB 1

## Maple Review

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*Topic 1: Working with MAPLE.*

- 1.) This is a Maple worksheet. It consists of text (like this), Maple commands (red with the prompt `>`) and Maple output (the blue).
- 2.) As you read along hit the enter key on each line with the prompt `>`. This will instruct Maple to carry out the command. This must be done every time you open up a previously written file. Maple does not "remember" the functions so it must redo the commands that it is supposed to know. This is especially important for function definitions.
- 3.) You can enter text at the prompt by clicking the "T" icon at the top of this window.
- 4.) You can get a new prompt by clicking the [`>`] icon.
- 5.) Every command ends with a semicolon (;) or colon(:). The difference is that after a colon Maple does not show any output. This is useful because Maple often prints information that you do not want to see.
- 6) You are asked to answer the questions in **boldface**. This will require that you execute some Maple commands and you interpret the results by writing what you noticed and replying to the questions.
- 7) If a command is entered and Maple does nothing after Enter is pressed or produced a message in red, there is a mistake. Possibly a semicolon is missing. If a command or expression is entered incorrectly, then click the mouse on the line to edit it and then re-

execute it by pressing Enter.

8) Maple can do arithmetic using +, -, \* for multiplication, / for division and ^ for exponentiation. To be safe, use parentheses to be sure that the operations are performed in the desired order. Use only round parentheses, not brackets.

```
> (3+4)/7;  
1  
-----  
> 3+4/7;  
25  
-----  
7
```

The results are not the same.

9) A typical mistake is forgetting the \* for multiplication:

```
> (2+5)(3-9);  
7
```

This cannot possibly be right:  $2+5=7$  while  $3-9=-6$ , so the product should be negative. The correct command is:

```
> (2+5)*(3-9);  
-42
```

### ***Topic 2: Decimal expansions***

If Maple starts with integers then it gives the answer in terms of integers whenever it can. If you want decimals either start with decimals or use the evalf command.

```
> 5/3; 5.0/3; evalf(5/3);  
5  
-----  
1.666666667  
1.666666667
```

A very important number in mathematics is pi.

The area of a disc of radius  $R$  is  $\pi R^2$ .

We will calculate  $\pi$  with many methods introduced in this course.

Every student of mathematics should compute some of its digits. Computing  $\pi$  has occupied mathematicians for many centuries. In Maple we use  $\text{Pi}$  to denote it. Pay attention to the capital P.

```
> Pi; evalf(Pi);
```

```
π  
3.141592654
```

```
> sqrt(2);
```

```
 $\sqrt{2}$ 
```

```
> sqrt(2.0);
```

```
1.414213562
```

You can also calculate square roots as the  $1/2$  power.

```
> 2^(1/2);
```

```
 $\sqrt{2}$ 
```

```
> sqrt(-1);
```

```
I
```

Remember that there is no real square root of  $(-1)$ . There is an imaginary square root for  $(-1)$  which is called  $i$  or  $I$ . *Do not use  $I$  for any other quantity in Maple.*

```
> sin(Pi/6);
```

```
 $\frac{1}{2}$ 
```

```
> sin(Pi/4); evalf(%);
```

```
 $\frac{1}{2}\sqrt{2}$   
0.7071067810
```

Another interesting number is  $e$ . It is easier to evaluate it using the exponential function, since  $e = \exp(1)$ .

```
> evalf(exp(1));  
2.718281828
```

Notice the use of %. Maple interprets this to be the last calculated answer.

```
> evalf(%);  
2.718281828
```

The evalf(%) redispayes the previous evalf(exp(1)).

Also notice that Maple uses radian when computing trigonometric numbers. For example 30 degrees equals  $\frac{\text{pi}}{6}$ .

**Write a formula that gives you an angle in radians, when you input it in degrees.**

**(Pi/180) RAD= DEG**

**Compute**  $\tan\left(\frac{\text{pi}}{2}\right)$ .

```
> tan( $\frac{\text{Pi}}{2}$ );  
Error, (in tan) numeric exception: division by zero
```

**What do you notice? It returns an error message "division by zero"**

**Topic 3: Defining a function.** To define  $f(x) = x^2$  in Maple you write the command

```
> f := x -> x^2;  
  
f := x -> x^2
```

Once you have defined the function you can use the usual notation to evaluate the function at points in the domain.

```
> f(5);  
  
25
```

```
> f(-3);
```

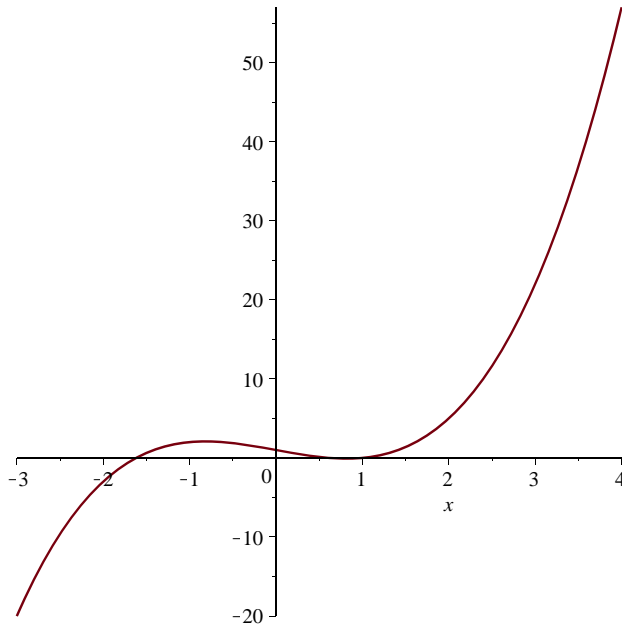
## Topic 4 The plot and fsolve commands

```
> f:= x-> x^3-2*x + 1;
```

$$f:=x \rightarrow x^3 - 2x + 1$$

To see the graph of f we use the plot command.

```
> plot(f(x), x = -3..4);
```



We can change the size of the graph by first clicking to it and then resizing it

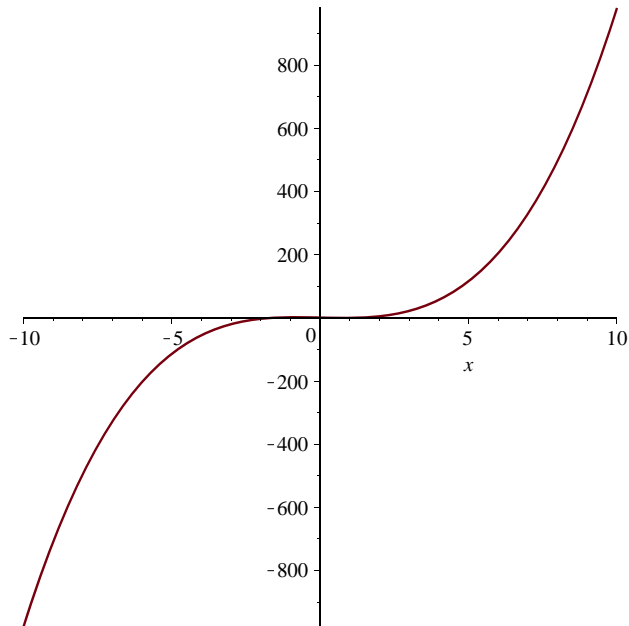
by clicking to one of its corners and dragging the mouse. Practice this in the graph above.

This graph is not detailed enough to show where it crosses the x-axis.

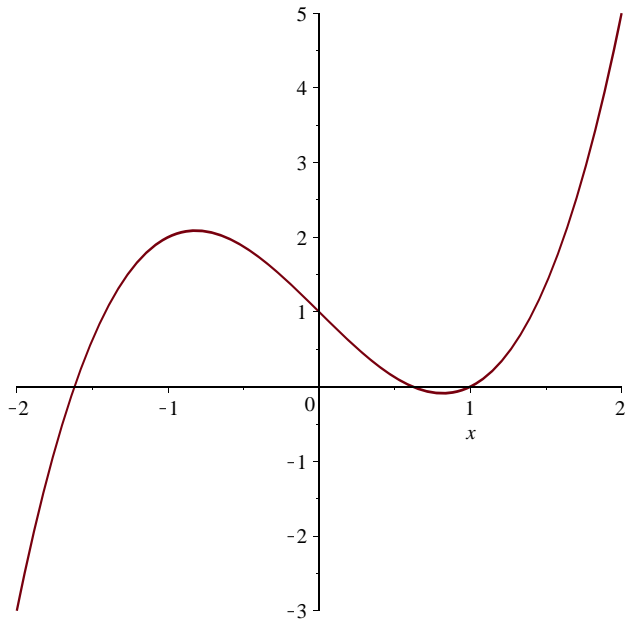
We can change the domain being graphed by changing the values of  $x$  in the plot command.

If no values are given for  $x$  then  $x=-10..10$  is assumed.

```
> plot(f(x),x);
```

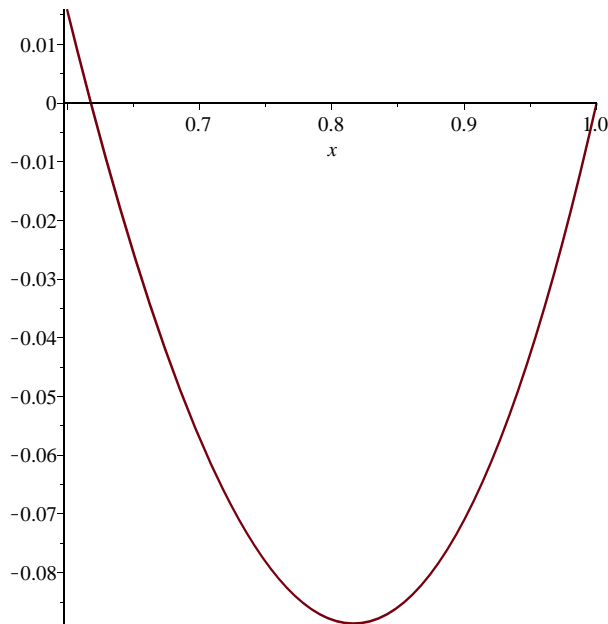


```
> plot(f(x), x=-2..2);
```



Let's look even closer to the right crossing.

```
> plot(f(x), x=0.6..1);
```



There appear to be two places between 0 and 1 where the graph crosses the  $x$ -axis. We can solve for decimal solutions using `fsolve`.

```
> fsolve(f(x) = 0,x=-2..-1);
```

```
-1.618033989
```

```
> fsolve(f(x) = 0,x=0.5..0.75);
```

```
0.6180339887
```

```
> fsolve(f(x) = 0,x=.75..1.25);
```

```
1.
```

Another command that works with polynomials is `solve`, instead of `fsolve`:

```
> solve(f(x)=0,x);
```



$$1, \frac{1}{2}\sqrt{5} - \frac{1}{2}, -\frac{1}{2} - \frac{1}{2}\sqrt{5}$$

```
> evalf(%);
```

The last command evaluates the three solutions and produces the same results as fsolve.

$$1., 0.6180339880, -1.618033988$$

Another command that is useful with polynomials is factor.

```
> factor(f(x));
```

$$(x-1)(x^2+x-1)$$

As you see Maple has identified the factor  $x-1$ , which gives as solution  $x=1$ . For the other two solutions, one can use the quadratic

formula to find the two solutions:  $r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$  and

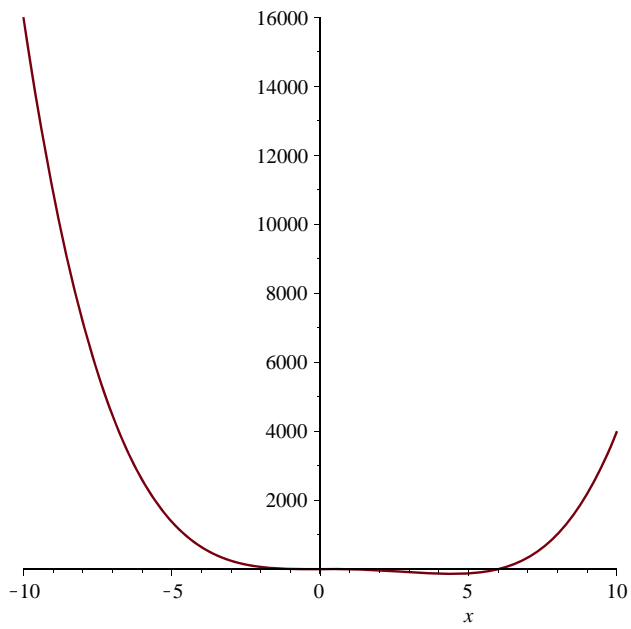
$$r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} .$$

Example : Find the range of  $g(x) = x^4 - 6x^3 + 1$ ;

```
> g := x->x^4-6*x^3+1;
```

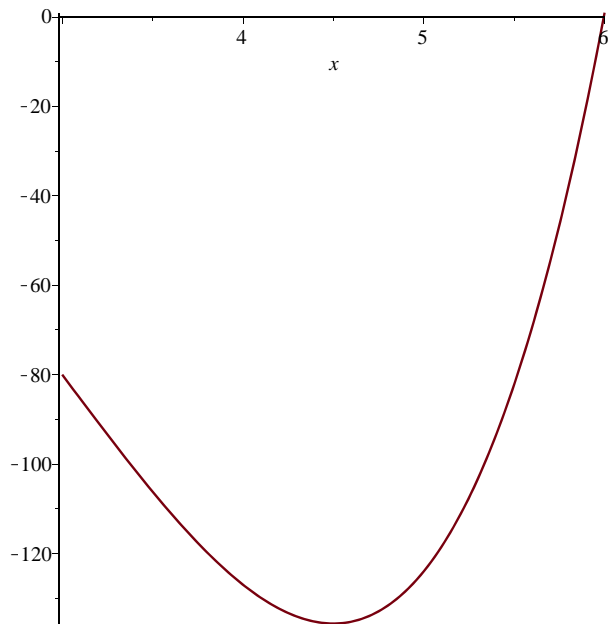
$$g := x \rightarrow x^4 - 6x^3 + 1$$

```
> plot(g(x),x);
```



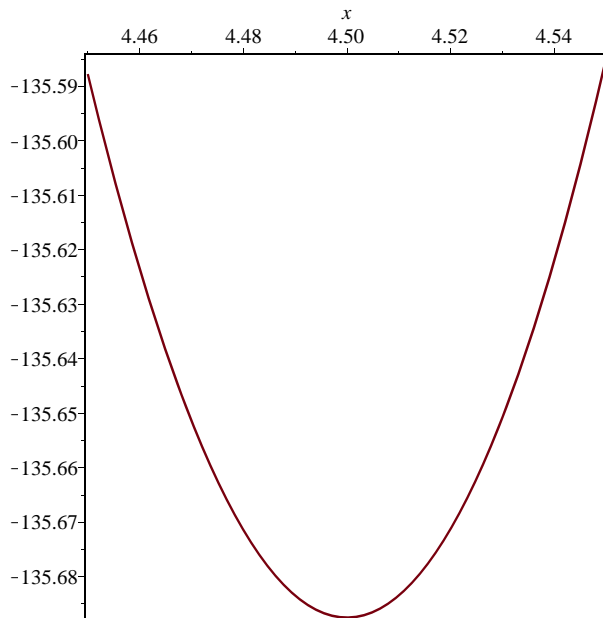
It appears that there is no maximum value but there is a minimum.  
To get an idea of what it is we must change the  $x$ -values in the plot.

```
> plot(g(x), x=3..6);
```



Now we zoom in a little more.

```
> plot(g(x), x=4.45..4.55);
```



```
> D(g);
```

$$x \rightarrow 4x^3 - 18x^2 \quad (1)$$

```
> fsolve(4x^3 - 18x^2 = 0, x);
```

$$0., 0., 4.500000000 \quad (2)$$

```
> g(4.5);
```

$$-135.6875 \quad (3)$$

So the range is about  $[-135.68, \textit{infinity})$ . With the techniques you studied in Calculus I, you can actually find the exact minimum.

Sometimes it is useful to plot two or more graphs at the same time.

To do

this we put the functions we want to graph into a set by putting  $\{ \}$  around them and then plot the set.

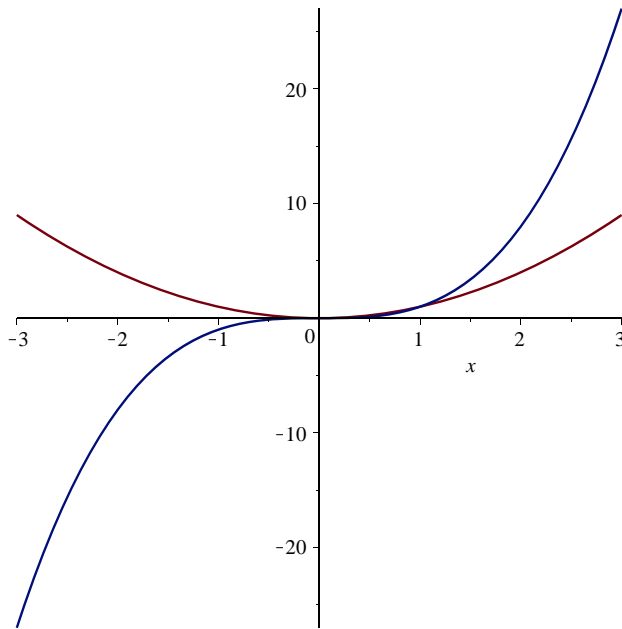
```
> f1 := x->x^2;
```

```
f1 := x -> x^2
```

```
> f2 := x -> x^3;
```

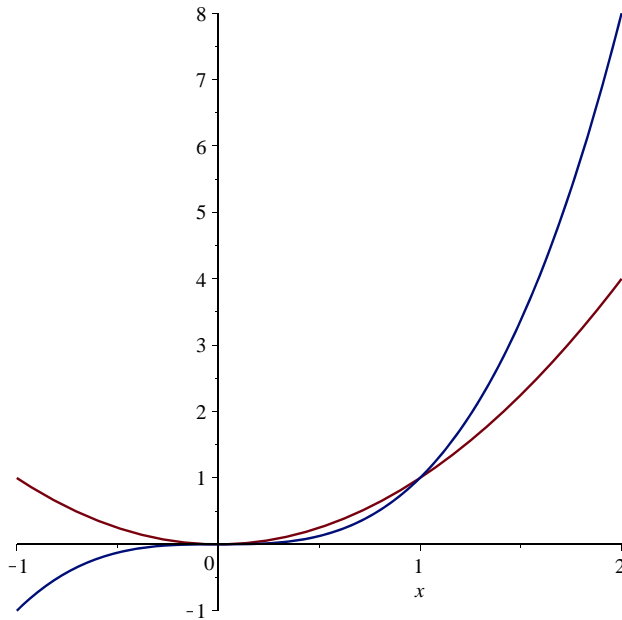
```
f2 := x -> x^3
```

```
> plot({f1(x), f2(x)}, x=-3..3);
```



As you see, the graphs are not very clear in the interval  $[-1, 1]$ . We can zoom in with

```
> plot({f1(x), f2(x)}, x=-1..2);
```



**When is  $f1(x)$  larger than  $f2(x)$ ? When is it smaller? Write a complete statement.**

`> fsolve(f1(x) - f2(x) = 0, x);`

`0., 0., 1.`

**(4)**

So one can conclude that  $f1(x)$  is greater than  $f2(x)$  on  $(0,1)$ , and  $f2(x)$  is greater than  $f1(x)$  otherwise.

**Example:** Graph on the same graph the functions  $g1(x)=x^5$  and  $g2(x)=20x^4$ .

We see that  $g2(x)$  is larger than  $g1(x)$  on this graph. Is this always the case?

**If not, how will you produce a better graph?**

`> g1 := x -> x^5;`

`g1 := x -> x^5`

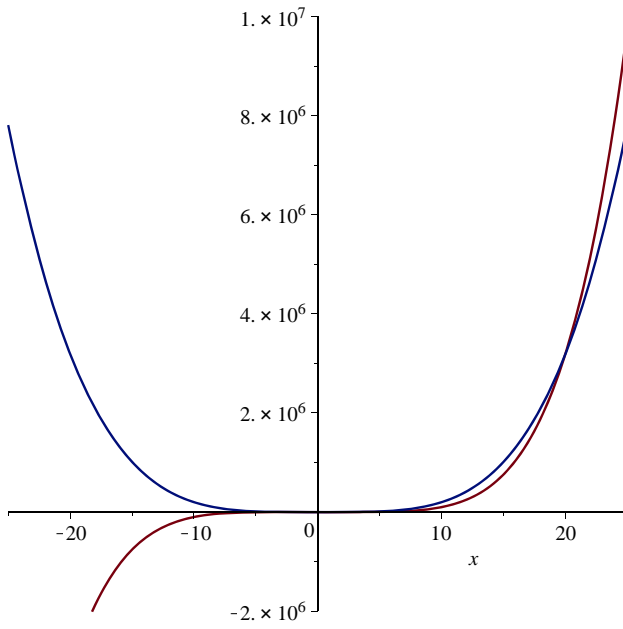
**(5)**

`> g2 := x -> 20 x^4;`

`g2 := x -> 20 x^4`

**(6)**

`> plot( {g1(x), g2(x)}, x=-25..25);`



```
> fsolve(g1(x) - g2(x) = 0, x);
```

```
0., 0., 0., 0., 20.
```

(7)

```
>
```

Exercise: For the above  $g(x)$ , using calculus idea and MAPLE, find out  $x$ -value which makes  $g(x)$  minimized.