MATH 156 LAB 4

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In this lab we start with the observation that $\displaystyle\int_{-1}^{1} \sqrt{1 - x^2}\ dx = \frac{Pi}{2}$.

This is so, because the integrand is the equation of a circle:

If we square y=$\sqrt{1 - x^2}$,
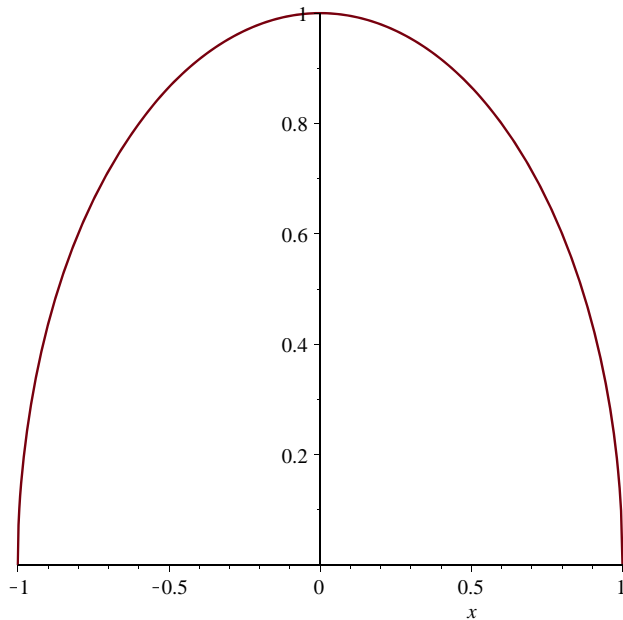
we get $y^2 = 1 - x^2$,

which gives $x^2 + y^2 = 1.$

Let us graph the integrand:

```
> f:=x->sqrt(1-x^2);
```

$$f := x \rightarrow \sqrt{1 - x^2}$$

```
> plot(f(x), x=-1..1);
```

This does not look like a (semi)circle. The reason is that Maple arranges the size of the axes to be pleasing to the eye, not to be "mathematically" correct.
To remedy the situation, we have to introduce the command "scaling=constrained", which tells Maple to use the same scale on the x and on the y axes.
This way there will be no distortion.

```
> plot(f(x), x=-1..1, scaling=constrained);
```
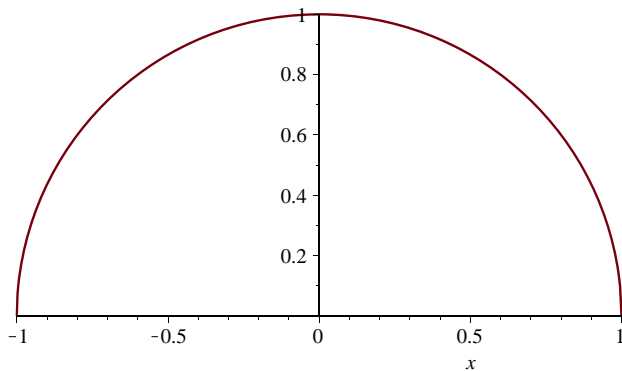
If you want to see the lower semicircle, we have to introduce the function $g(x) = -\sqrt{1 - x^2}$.

**Introduce a command that defines this function.**

```
> g:=x->-sqrt(1-x^2);
```

$$g := x \rightarrow -\sqrt{1 - x^2}$$

We would not only like to plot this function, but also see the two plots together. Introduce commands that plot $g(x)$, **name this plot and the plot for** $f(x)$ **and display the two plots simultaneously. Do not forget to introduce the plots package.**
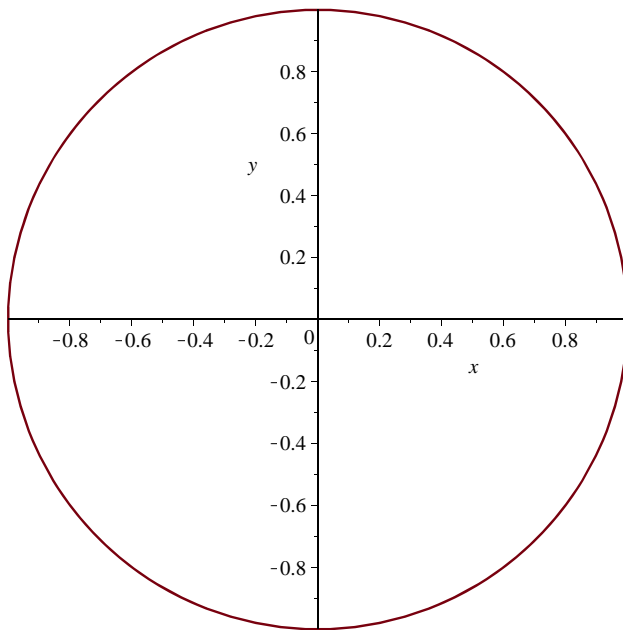
```
> with(plots):
> display(plot(f(x), x=-1..1, scaling=constrained),plot(g(x), x=-1.
  .1, scaling=constrained));
```
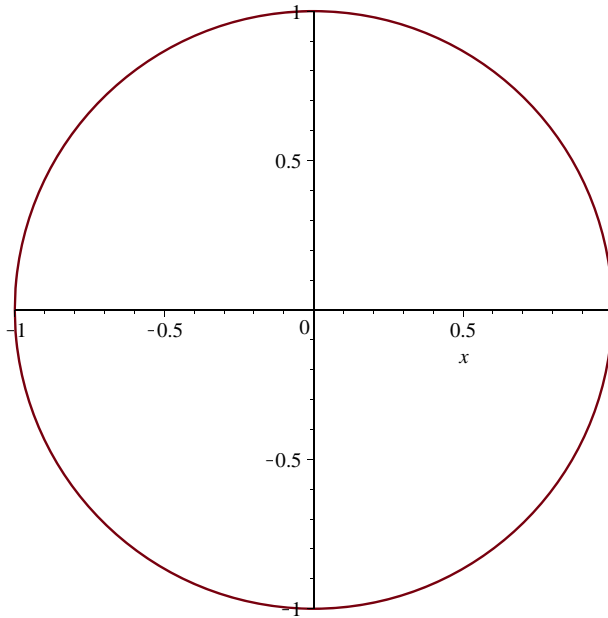
This process is rather cumbersome. Maple has another command that will plot for us the circle with equation $x^2 + y^2 = 1$ in an easier way: This is the implicitplot command:
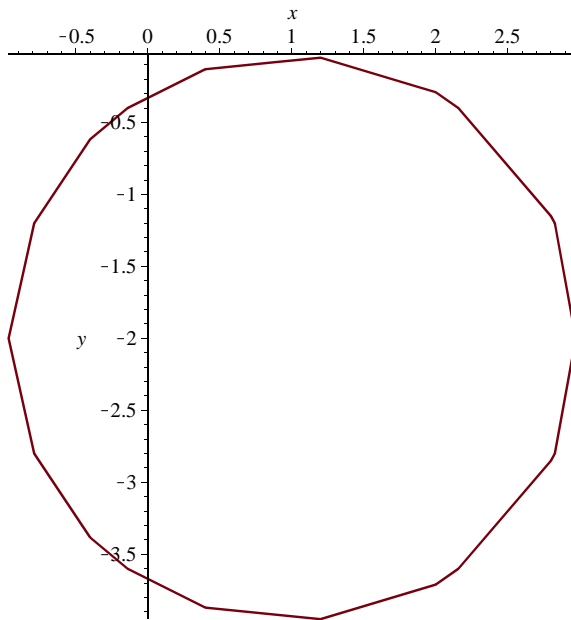
```
> implicitplot(x^2+y^2=1, x=-1..1, y=-1..1, scaling=constrained);
```

Notice that we also gave the range of y-values.

Plot the equation $(x-1)^2 + (y+2)^2 = 4$. How do you decide on the range of x and y-values?

```
> implicitplot((x-1)^2+(y+2)^2=4, x=-10..10, y=-10..10, scaling=
  constrained);
```

This is the equation of a circle with radius 2 and center (1, -2). In general the equation of a circle with radius R and center at (a, b) is

$$(x - a)^2 + (y - b)^2 = R^2.$$

The first plot we saw today looked like an elongated circle. This has the name ellipse in mathematics. An ellipse centered at the origin has an equation of the form

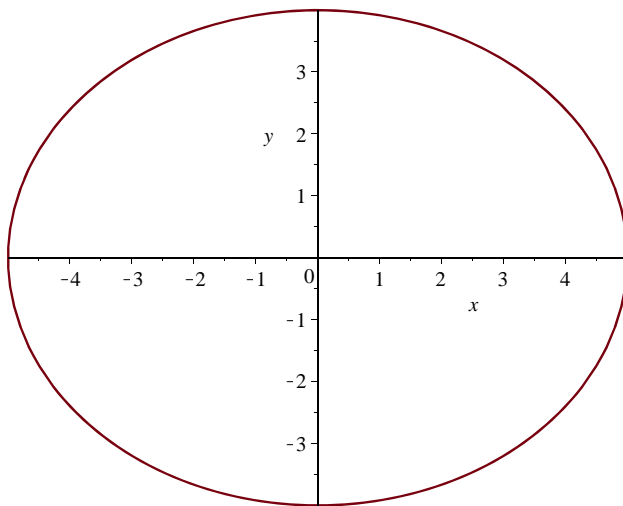$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$ The x-intercepts of this equation are (a, 0), (−a, 0).

The y-intercepts of the ellipse are (0, b), (0, −b). The numbers a and b are called the lenghts of the semiaxes. The largest of the two semiaxes is the major semiaxis and the smallest is the minor
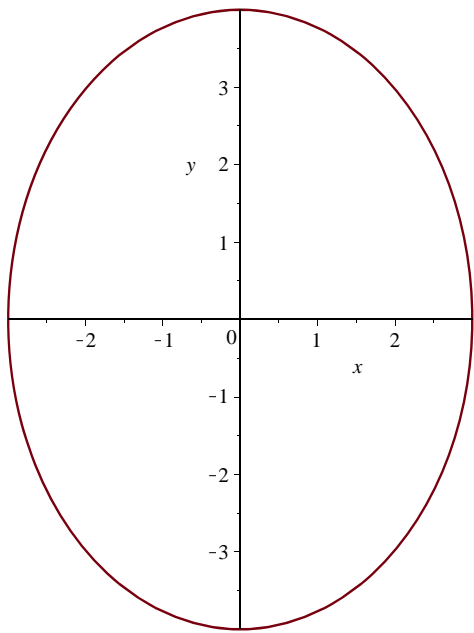
semiaxis.

Let us plot two ellipses with equations: $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ and $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$.

```
> implicitplot(x^2/25+y^2/16=1, x=-5..5, y=-4..4, scaling=
  constrained);
```



```
> implicitplot(x^2/9+y^2/16=1, x=-3..3, y=-4..4, scaling=
  constrained);
```

How do you decide on the range on the x and y values? What do you notice?

If you wonder why ellipses are important, the answer comes from astronomy: The trajectories of planets and satellites are ellipses. If the center of the ellipse is at the point (c, d) and the semiaxes have length a, b along the x, y axes, then the equation of the ellipse is:

$$\frac{(x-c)^2}{a^2} + \frac{(y-d)^2}{b^2} = 1.$$ **Plot the ellipse with equation**

$$\frac{(x-2)^2}{16} + \frac{(y-1)^2}{9} = 1.$$

Sometimes we are given other equations that represent circles and ellipses but it is a bit harder to see what the geometric data of the plot are.

For instance, the ellipse you have just plotted can be given by the equation: $9\,x^2 + 16\,y^2 - 36\,x - 32\,y = 92$. You can verify this by using the simplify command:

```
> simplify((x-2)^2/16+(y-1)^2/9);
```

$$\frac{1}{16}\,x^2 - \frac{1}{4}\,x + \frac{13}{36} + \frac{1}{9}\,y^2 - \frac{2}{9}\,y$$

This just expands the left-hand side. To clear the denominators, we multiply with 16 X 9=144.

```
> %*144;
```

$$9\,x^2 + 16\,y^2 - 36\,x - 32\,y + 52$$

It is easy to check that $144 - 52 = 92$.

```
> 144-52;
```

$$92$$

It would be nice if we could start from the equation $9\,x^2 + 16\,y^2 - 36\,x - 32\,y = 92$ and see the equation of the ellipse in

the form $\dfrac{(x-2)^2}{16} + \dfrac{(y-1)^2}{9} = 1.$

For this we need to complete the square in the equation. First of all we give a name to the left-hand side and introduce the student package:

```
> left:=9*x^2-36*x+16*y^2-32*y; with(student):
```

$$left := 9\,x^2 + 16\,y^2 - 36\,x - 32\,y$$

```
> newleft:=completesquare(left, {x, y});
```

$$newleft := 16\,(y-1)^2 - 52 + 9\,(x-2)^2$$

The $\{x, y\}$ tells Maple to complete the square in both the x, y variables. Now we move -52 to the right-hand side:

```
> right:=52+92;
```

$$right := 144$$

```
> finalleft:=newleft+52;
```
$$finalleft := 16\,(y-1)^2 + 9\,(x-2)^2$$

```
> finalleft/right;
```
$$\frac{1}{16}\,(x-2)^2 + \frac{1}{9}\,(y-1)^2$$

```
> eqn:=%=1;
```
$$eqn := \frac{1}{16}\,(x-2)^2 + \frac{1}{9}\,(y-1)^2 = 1$$

Complete the square and identify the important geometric data for the equation $x^2 + y^2 - 6\,x + 4\,y = 3$. Plot the equation.

```
> left:=x^2+y^2-6*x+4*y;
```
$$left := x^2 + y^2 - 6\,x + 4\,y$$

```
> newleft:=completesquare(left,{x,y});
```
$$newleft := (y+2)^2 - 13 + (x-3)^2$$

```
>  newleft + 13
   ─────────── ;
        16
```
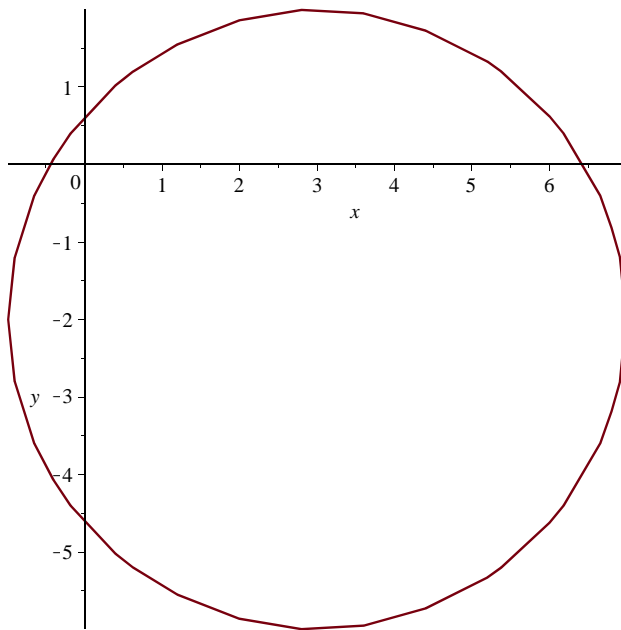$$\frac{1}{16}\,(y+2)^2 + \frac{1}{16}\,(x-3)^2 \qquad\qquad \textbf{(1)}$$

```
> eqn := % = 1;
```
$$eqn := \frac{1}{16}\,(y+2)^2 + \frac{1}{16}\,(x-3)^2 = 1 \qquad\qquad \textbf{(2)}$$

```
> implicitplot(eqn, x = -10..10, y = -10..10, scaling = constrained);
```

To verify the answer we also plot the original equation with the same ranges and we get the same plot:

```
> implicitplot(x^2+y^2-6*x+4*y = 3, x=-1..7, y=-6..2, scaling=
  constrained);
```
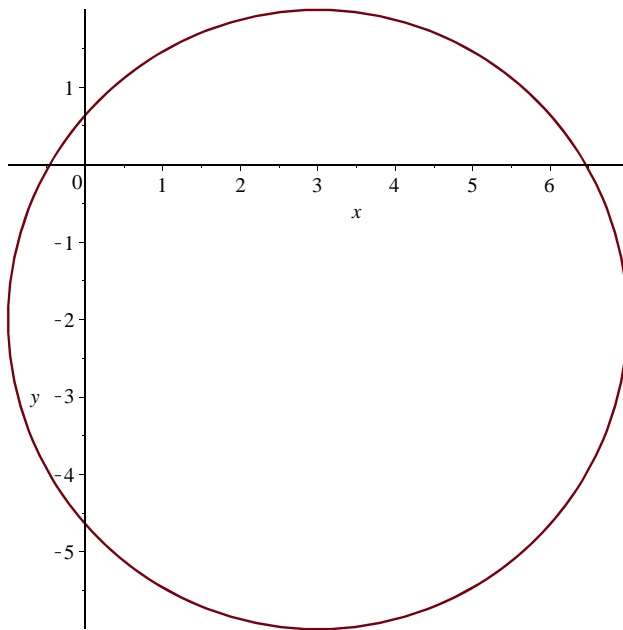
Identify the equation $9\,x^2 + 4\,y^2 + 36\,x = 0$. Plot it directly and after completing the square.

```
> left:=completesquare(9*x^2+4*y^2+36*x,{x,y});
```
$$left := 9\,(x+2)^2 - 36 + 4\,y^2$$

```
> left/36;
```
$$\frac{1}{4}\,(x+2)^2 - 1 + \frac{1}{9}\,y^2$$

```
> eqn := left/36 + 1 = 1;
```
$$eqn := \frac{1}{4}\,(x+2)^2 + \frac{1}{9}\,y^2 = 1 \tag{3}$$

```
> implicitplot(eqn, x = -10..10, y = -10..10, scaling = constrained);
```