Midterm Exam

Spring 2014, MAT156 Section 03LB[51316]

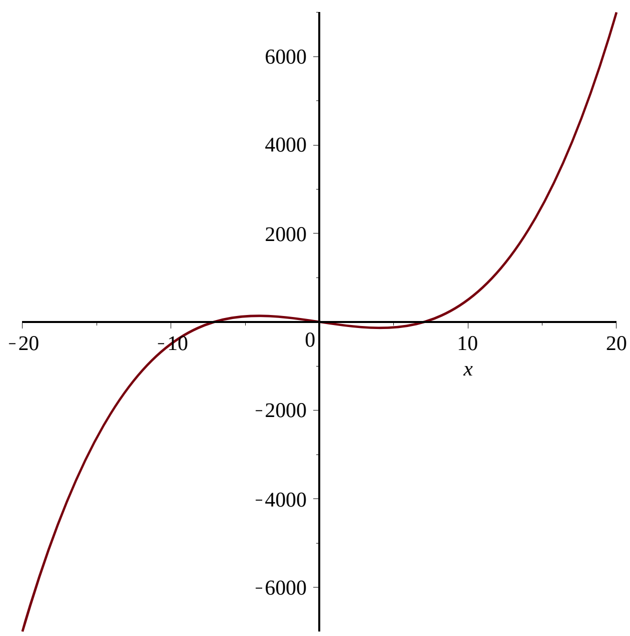
March 4th, 2014. 11:00AM--12:40PM.

Instructions: You can use only MAPLE program and a web browser(only for the purpose of submitting your exam solution to the instructor). You may not use any other programs other than these two. You can look up your MAPLE source files, but otherwise this exam is closed-book, closed-note, and you may not use any electronic device in this exam except your PC. You are not allowed to talk to other students. Type all details explicitly. All solutions should be obtained by using MAPLE codes.

Problem 1. Consider the function f(x)=x^3-50*x + 1 (30 Points. 5 Points each)
(1) Plot the graph of f(x) so that the graph will show all intersections with x-axis.
(2) Using MAPLE, find all real roots.

```
> f:=x -> x^3-50*x + 1; plot(f(x), x=-20..20);
```

$$f := x \rightarrow x^3 - 50\,x + 1$$



```
> fsolve(f(x));
```

$$-7.081046678, 0.02000016000, 7.061046518 \quad (1)$$

```
>
```

(3) Using MAPLE, find the derivative of f(x).
(4) Using MAPLE, find all critical numbers of f(x).
(5) Find relative extremums.

```
> diff(f(x),x);
```

$$3x^2 - 50 \quad (2)$$

```
> fsolve(%=0,x);
```

$$-4.082482905, 4.082482905 \quad (3)$$

```
> f(-4.082482905); f(4.082482905);
```

$$137.0827634$$

$$-135.0827634 \quad (4)$$

(6) Find the second derivative of f(x) as a function(and not an expression!). Using MAPLE, find all inflection points.

```
> D(D(f));
```

$$x \rightarrow 6x \quad (5)$$

```
> fsolve(6*x=0);
```

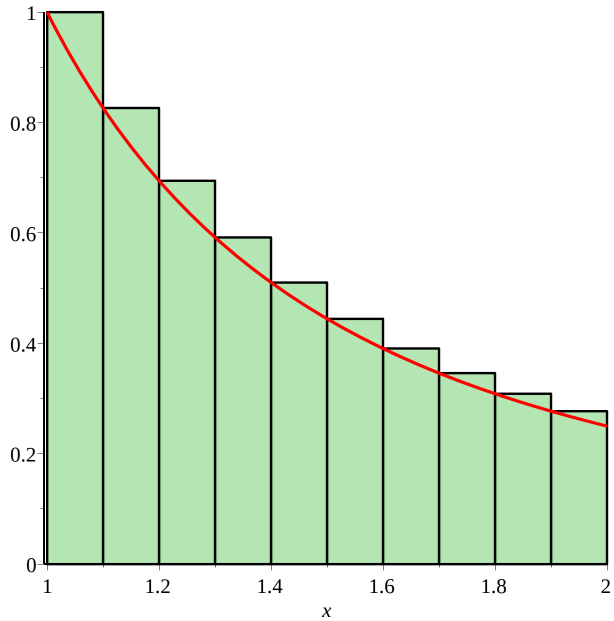
$$0. \quad (6)$$

Problem 2. (40 points. 5 points each except (7)) Let g(x)=1/x^2. Consider the interval [1,2]. Run the student package using "with(student):"
(1) Show graphically the left hand sum for 10 subintervals. Give it a name.
(2) Show graphically the right hand sum for 10 subintervals. Give it a name.
(3) Show graphically the midpoint rule for 10 subintervals. Give it a name.
(4) Using "with(plots):", display the above three pictures in a single graph.
(5) Give a numerical value(in decimal expression) for left hand sum, midpoint rule, and right hand sum for 10 subintervals.
(6) Using MAPLE, calculate the integral of g(x) from 1 to 2. Explain, for the result from (5), which one is overestimating and underestimating.
(7) Using a loop, write commands that calculate the trapezoid rule, midpoint rule, left-hand sums and right-hand sums with 5, 10, 20, 40, 80, 160, 320, 640, 1280, 2560 subintervals. (10 points)

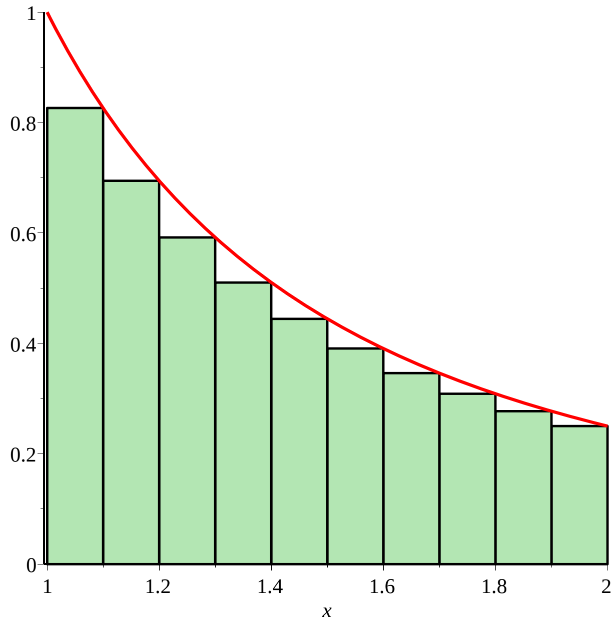
```
> with(student): g:=x->1/x^2;
```

$$g := x \rightarrow \frac{1}{x^2} \quad (7)$$

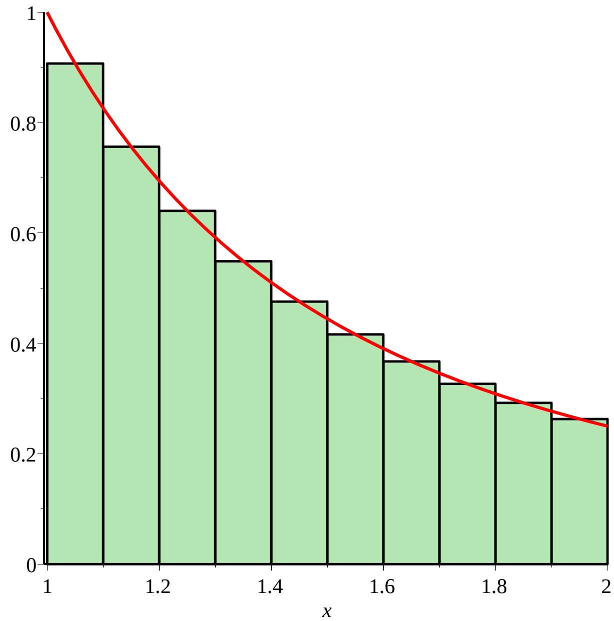
```
> leftbox(g(x),x=1..2,10); rightbox(g(x),x=1..2,10); middlebox(g
  (x),x=1..2,10);
```



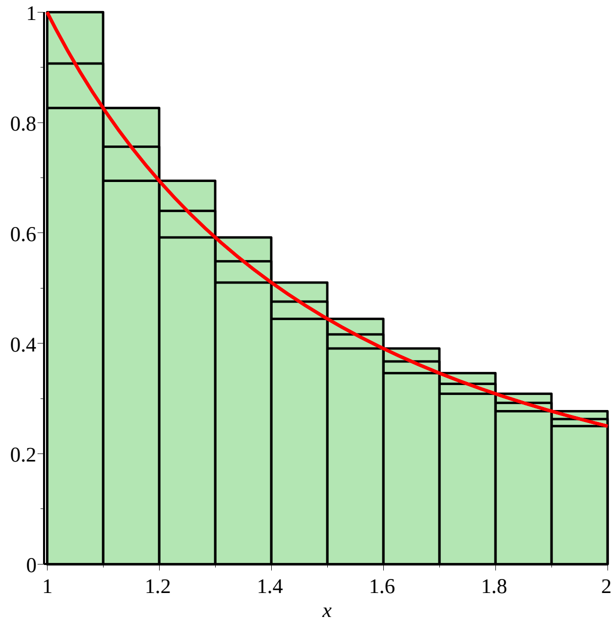
1
0.8
0.6
0.4
0.2
0
1
1.2
1.4
1.6
1.8
2
x



1
0.8
0.6
0.4
0.2
0
1
1.2
1.4
1.6
1.8
2
x




```
> lhs10:=leftbox(g(x),x=1..2,10): rhs10:=rightbox(g(x),x=1..2,10):
  mids10:=middlebox(g(x),x=1..2,10):
> with(plots): display(rhs10,mids10,lhs10);
```




```
> evalf(leftsum(g(x),x=1..2,10)); evalf(middlesum(g(x),x=1..2,10));
  evalf(rightsum(g(x),x=1..2,10));
```

0.5389551275

0.4992736363

0.4639551275 **(8)**

```
> evalf(Int(g(x),x=1..2));
```

0.5000000000 **(9)**

Hence the left hand sum is overestimating, and the middlesum and the rightsum is underestimating.

```
> for k from 0 to 9 do N := 5*2^k; evalf(leftsum(g(x),x=1..2,N));
  evalf(.5*(leftsum(g(x),x=1..2,N)+rightsum(g(x),x =1..2,N)));
  evalf(rightsum(g(x),x=1..2,N)) od;
```

$N := 5$

0.5807831002

0.5057831002

0.4307831002

$$N := 10$$

0.5389551275

0.5014551276

0.4639551275

$$N := 20$$

0.5191143820

0.5003643820

0.4816143819

$$N := 40$$

0.5094661332

0.5000911332

0.4907161332

$$N := 80$$

0.5047102856

0.5000227856

0.4953352856

$$N := 160$$

0.5023494466

0.5000056966

0.4976619466

$$N := 320$$

0.5011732991

0.5000014240

0.4988295491

$$N := 640$$

0.5005862936

0.5000003561

0.4994144186

$$N := 1280$$

0.5002930577

0.5000000890

0.4997071202

$$N := 2560$$

0.5001465066

0.5000000222

0.4998535379 **(10)**

Problem 3. (15 points, 5 points each) Identify the equation 9x^2+4y^2-18x-8y-23=0.
(1) Complete the square.
(2) Determine what kind of conic section the given equation defines among circle, a parabola, a hyperbola, an ellipse, or two straight lines. If it is a circle or an ellipse, specify the radii and the center, and if it is a parabola or a hyperbola, specify the focal points.
(3) Plot the graph, by specifing the ranges of x and y values.

```
> completesquare(9*x^2+4*y^2-18*x-8*y-23,{x,y});
```

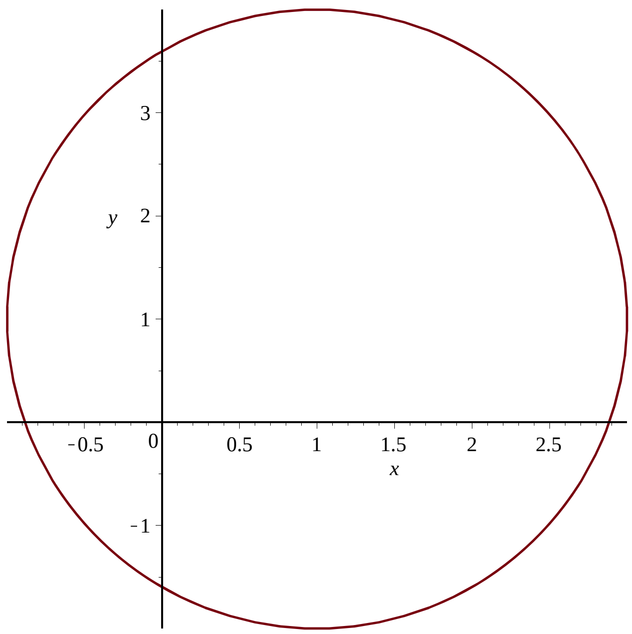
$$4\,(y-1)^2-36+9\,(x-1)^2 \quad \textbf{(11)}$$

```
> eqn:=%/36;
```

$$eqn := \frac{1}{9}\,(y-1)^2-1+\frac{1}{4}\,(x-1)^2 \quad \textbf{(12)}$$

From this we see the given equation defines an ellipse with its center (1,1), the long axis length 4, and the short axis length 6.

```
> implicitplot(eqn=0,x=-1..3, y=-2..4);
```



Problem 4. Graph a half-circle with a radius 5 and the center (1,0), in a way that the graph that MAPLE shows really looks like a half-circle, which is not distorted like an half ellipse. (5 Points)

```
> plot(sqrt(25-(x-1)^2),x=-4..6, scaling=constrained);
```