

MAT175 Departmental Final Exam Practice Test
Spring 2014, MAT175 Section B401[51350]

Instructions: This exam should be taken without text or notes or electronic devices. Show your work, and indicate answers clearly. Cross out all work that you do not want to be graded.

1. Compute the derivative $\frac{dy}{dx}$ for $y = \frac{1}{\sqrt{\pi e}} - \frac{7}{x^4} + 5\sqrt[3]{x}$.(5 Points)
2. Compute the derivative $p'(y)$ of the function $p(y) = \frac{7y-2}{5y+7}$.(5 Points)
3. Write down an equation of the tangent line to the graph of $y = 3 + 2x^2 + 5e^x$ at the point where $x = 0$.(5 Points)
4. Determine the slope of the tangent line to the graph of the equation $2x^2 + 5y^2 + 2xy = 17$ at the point $(2, 1)$.(5 Points)
- 5 Compute the derivative $\frac{dw}{dz}$ of the function $w = \ln(1 - \cos ze^z)$.(5 Points)
6. Evaluate the following limit: (5 Points)

$$\lim_{\theta \rightarrow 0} \frac{3 \sin 4\theta \cos(\theta + \frac{\pi}{4})}{5\theta}.$$

7. Evaluate the following limit: (5 points)

$$\lim_{x \rightarrow 2} \frac{x^2 - 7x - 10}{x^2 - 4}.$$

8. Find the limit: (5 Points)

$$\lim_{x \rightarrow \infty} \frac{6x^4 + 100}{7x^4 - 100}.$$

9. For which constant k is the following function $Q(x)$ continuous for all x ? Justify your answer.(5 Points)

$$Q(x) = \begin{cases} 3 \sin x + ke^x & \text{if } x \leq 0 \\ 5 + \cos x + 7 & \text{if } 0 < x \end{cases}$$

10. Show that the derivative of $f(x) = x + 3x^2$ is $f'(x) = 1 + 6x$ by using the definition of the derivative as the limit of a difference quotient.(10 Points)

11. If the area $A = s^2$ of an expanding square is increasing at the constant rate of 120 square inches per second, how fast is the length s of the sides increasing when the area is 16 square inches?(5 Points)
12. Find where the graph of $y = \frac{7}{6}x^3 - \frac{7}{2}x^2 - \frac{7}{6}x + \frac{7}{2}$ is concave up and concave down, and find all inflection points.(5 Points)
13. Find and classify all relative extrema of $F(x) = x^4 - 4x^2 + 2$.(5 Points)
14. Find the absolute maximum and minimum values of $f(x) = -\frac{2}{3} + \frac{2}{3}x^3 - \frac{1}{2}x^4$ on the closed interval $[0, 1]$.(8 Points)
15. A particle moves along the x -axis with an acceleration given by $a(t) = 3t+1$, where t is measured in seconds and s (position) is measured in meters. If the initial position is given by $s(0) = 3$ and the initial velocity is given by $v(0) = 4$ then find the position of the particle at t seconds.(8 Points)
16. Find the area under the curve $y = 6 - 3x^2$ from $x = 0$ to $x = 1$. (5 Points)
17. Evaluate the derivative $F'(x)$ of the function $F(x)$ defined by:

$$F(x) = \int_0^x \frac{t}{\sqrt{1+t^4}} dt$$

at $x = 3$.

18.(6 Points) SOLVE **ONLY ONE** OF THE FOLLOWING TWO PROBLEMS:

- (a) In the sales of a math textbook, it is known that the unit price x of the textbook and the number of books sold $N(x)$ has the following relation: $N(x) = 20 - 2x$. What value of x maximizes the the total sales amount?
- (b) Evaluate $\int x^2(2x^3 + 3)^7 dx$.