

Midterm Exam I

Spring 2014, MAT175 Section B401[51350]

March 13th, 2014. 9:00AM–10:40AM.

Instructions: Print your name on the exam booklet. This exam is closed-book and closed-note. You cannot use any electronic device in this exam. You are not allowed to talk to other students. Write all details explicitly. Answers without justifications and/or calculation steps may receive no score. Hand-in blue booklet only, and keep the exam paper for your study.

Part I — 5 points each, total 55 points

1.(Sample Final I-9) Evaluate the following limit:

$$\lim_{x \rightarrow 6} \frac{2x + 1}{\sqrt{x + 3}}.$$

2.(Sample Final I-8) Evaluate the following limit:

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}.$$

3. Evaluate the following limit:

$$\lim_{x \rightarrow 3} \frac{\sqrt{x + 1} - 2}{x - 3}.$$

4.(Sample Final II-7 variant) Evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}.$$

5. (Sample Final I-16) Find the constant a such that the function is continuous on the entire real number line.

$$f(x) = \begin{cases} x^3 & \text{if } x \leq 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

6. (5 points) Evaluate the following limit:

$$\lim_{x \rightarrow e} (\ln x^2 + 2^{x/e})$$

7. (5 points) Find all vertical asymptotes of the following:

$$f(x) = \frac{x + 1}{x^2 - 1}.$$

8.(Sample Final I-1) Compute the derivative $\frac{dy}{dx}$ for $y = e^2 + \frac{1}{e} + 3e^x + 2 \ln x$.(5 Points)

9.(Sample Final I-2) Compute the derivative $p'(c)$ of the function $p(c) = \pi c \cos(\pi x) + \frac{x}{c} + c + ce^x$. (5 Points) *Hint: What is the variable? What are constants?*

10.(Sample Final I-3) Write down an equation of the tangent line to the graph of $y = x + \cos x$ at the point where $x = 0$.

11.(Sample Final I-12) If the position of a particle moving in a straight line is given by $x(t) = t^4 + 2t$ after t seconds, find *both* the velocity *and* the acceleration of the particle when $t = 1$.

Part II — points varies, total 45 points

12.(Sample Final I-15) Show that the derivative of $f(x) = 2x^2 - 1$ is $f'(x) = 4x$ by using the definition of the derivative as the limit of a difference quotient.(10 Points)

13. (10 points) By using the squeeze theorem(i.e. the sandwich lemma), show that:

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0.$$

14. (10 points) For the function $f(x) = \frac{|x|}{x}$, discuss existence of limit as x approaches to 0.

15. (10 points) Using the intermediate value theorem, show that $f(x) = x^3 - 3x$ has at least one zero in the interval $[1, 4]$.(No point will be given if you do not use the intermediate value theorem)

16. (5 points) Prove or disprove the following statement: “Any continuous function defined on \mathbb{R} is differentiable.”