

Midterm Exam I Practice Test

Spring 2014, MAT175 Section B401[51350]

Actual Midterm Date and Time: March 13th, 2014. 9:00AM–10:40AM.

Instructions: Write all details explicitly. Answers without justifications and/or calculation steps may receive no score.

Part I — 5 points each, total 50 points

1.(Sample Final I-9) Evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{2014x + 10}{8x^{2013} + 5}.$$

2.(Sample Final 7) Evaluate the following limit: (5 points)

$$\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 5x + 6}.$$

3. Evaluate the following limit:

$$\lim_{x \rightarrow 1} \frac{\sqrt{2x-1} - \sqrt{x}}{x-1}.$$

4.(Sample Final 6) Evaluate the following limit:

$$\lim_{x \rightarrow 1} \frac{2 \tan(x-1)}{7(x-1)}.$$

5. (Sample Final 10) Find the constant a such that the function is continuous on the entire real number line.

$$f(x) = \begin{cases} \frac{x^2 - a^2}{x - a} & \text{if } x \neq a \\ 16 & \text{if } x = a \end{cases}$$

6. (5 points) Evaluate the following limit: (*Hint*: $\cos \pi = -1$)

$$\lim_{\phi \rightarrow \pi} \phi \cos \phi.$$

7. (5 points) Let $f(x) = x^2$. Calculate:

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

8.(Sample Final I-1) Compute the derivative $\frac{dy}{dx}$ for $y = \sqrt{7} + \frac{1}{x^3} + 2\sqrt{x}$.(5 Points)

9.(Sample Final I-2) Compute the derivative $f'(t)$ of the function $f(t) = \frac{x^2}{t^2} + \frac{t^2}{x^2} + tx$.(5 Points)
Hint: What is the variable? What are constants?

10.(Sample Final I-3) Write down an equation of the tangent line to the graph of $y = e^{2x} + 2x$ at the point where $x = 0$. Hint: $(e^{2x})' = 2e^{2x}$. (5 Points)

11.(Sample Final I-12) If the position of an object dropped from a height of 64m is given by $h(t) = 64 - \frac{1}{2}gt^2$ after t seconds, where $g = 9.8m/s^2$ is the gravitational acceleration, find *both* the velocity *and* the acceleration of the object when it hits the ground. Note: It is not required to substitute g into a number. One can leave g in the answer.(5 Points)

Part II — points varies, total 50 points

12.(Sample Final I-15) Show that the derivative of $f(x) = x^2 + x$ is $f'(x) = 2x + 1$ by using the definition of the derivative as the limit of a difference quotient.(10 Points)

13. (10 points. This will be DEFINITELY on the actual exam) By using the squeeze theorem(i.e. the sandwich lemma), show that:

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0.$$

14. (10 points. This will be DEFINITELY on the actual exam) For the function $f(x) = \frac{|x|}{x}$, discuss existence of limit as x approaches to 0.

15. (10 points. This will be DEFINITELY on the actual exam) Using the intermediate value theorem, show that $f(x) = x^3 - 3x$ has at least one zero in the interval $[1, 4]$.(No point will be given if you do not use the intermediate value theorem)

16. (5 points. This will be DEFINITELY on the actual exam) Prove or disprove the following statement: “Any continuous function defined on \mathbb{R} is differentiable.”