Midterm Exam I Solution MAT 175 Section B401 Spring 2014.

#1. $\lim_{x\to 6} \frac{2x+1}{\sqrt{x+3}} = \frac{2\cdot 6+1}{\sqrt{6+3}} = \underline{\underline{\frac{13}{3}}}$.

#2. $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \lim_{x\to 2} x+2 = \underline{\underline{4}}$.

#3. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)}$

$= \lim_{x\to 3} \frac{\cancel{x+1-4}}{\cancel{(x-3)}(\sqrt{x+1}+2)} = \underline{\underline{\frac{1}{4}}}$.

#4. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \frac{\sin 2x}{\sin 3x} \cdot \frac{3x}{2x} \cdot \frac{2x}{3x}$

$= \lim_{x\to 0} \frac{\frac{\sin 2x}{2x}}{\frac{\sin 3x}{3x}} \cdot \frac{2}{3} = \frac{\lim_{2x\to 0} \frac{\sin 2x}{2x}}{\lim_{3x\to 0} \frac{\sin 3x}{3x}} \cdot \frac{2}{3}$

As $x\to 0$, $2x\to 0$, $3x\to 0$

$= \underline{\underline{\frac{2}{3}}}$.

#5. Clearly $f(x)$ is continuous for all $x \neq 1$, for any given a. For $f(x)$ to be continuous at $x=1$ we need: $\boxed{\lim_{x\to 1} f(x) = f(1)}$ ⊛, and for this equality to make sense, we need the following: $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$ (Side limits must agree)

whereas $\lim_{x\to 1^+} f(x) = ax^2\big|_{x=1} = a$

$\lim_{x\to 1^-} f(x) = x^3\big|_{x=1} = 1$.

So $\boxed{a=1}$. We check ⊛:

when $a=1$, $\lim_{x\to 1} f(x) = 1 = f(1) (= 1^3)$. ✓

#6. $\lim_{x\to e}\left(\ln x^2 + 2^{x/e}\right) = \ln e^2 + 2^{e/e}$

$= 2\ln e + 2^1$

$= 2+2 = \underline{\underline{4}}$.

#7. $f(x) = \frac{x+1}{(x+1)(x-1)}$

Since the denominator and the numerator are continuous, and $x+1\big|_{x=1} = 2 \neq 0$, whereas $(x+1)(x-1)$ vanishes at $x=1$, $f(x)$ has a vertical asymptote at $x=1$

i.e. $\boxed{x=1}$ is a vertical asymptote.

Note: $x=-1$ is not a vertical asymptote:

$\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{1}{x-1} = -\frac{1}{2}$.

#8. $y = e^2 + \frac{1}{e} + 3e^x + 2\ln x$.

$y' = 0 + 0 + 3e^x + \frac{2}{x}$

So $\underline{\frac{dy}{dx} = 3e^x + \frac{2}{x}}$

#9. $p(c) = \pi c \cos \pi x + \frac{x}{c} + c + ce^x$

$\underline{p'(c) = \pi\cos\pi x - \frac{x}{c^2} + 1 + e^x}$.

#10. $y = x + \cos x$ at $x=0$

point: $(0, 1)$ Note $\cos(0)=1$

slope: $y'\big|_{x=0} = 1-\sin x\big|_{x=0} = 1$

$\Rightarrow y-1 = 1\cdot(x-0)$

$\underline{y = x+1}$

#11. $x(t) = t^4 + 2t$.

$v(t) = x'(t) = 4t^3+2$

$a(t) = v'(t) = x''(t) = 12t^2$.

$\Rightarrow v(1) = 6$

$a(1) = 12$

#12. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2(x+h)^2 - 1 - (2x^2 - 1)}{h}$

$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + h^2 \cancel{-1} \cancel{-2x^2} \cancel{+1}}{h}$

$= \lim_{h \to 0} 4x + h = 4x$. ✓

#13. From $-1 \leq \sin\frac{1}{x} \leq 1$

it follows that $-x \leq x\sin\frac{1}{x} \leq x$ for all $x \neq 0$.

Since $\lim_{x \to 0} -x = 0 = \lim_{x \to 0} x$,

by the Squeeze theorem,

$\lim_{x \to 0} x \sin\frac{1}{x} = 0$. ✓

#14. $\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = +1$

$\neq$

$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = -1$.

Since the left and right limit disagree,

$\lim_{x \to 0} \frac{|x|}{x}$ does not exist. ✓

#15. First observe that $f(1) = -2$ $f(2) = 2$.

Since $f(x)$ is a continuous function on a closed interval $[1, 2]$, by the Intermediate value theorem, there exists some $c \in [1, 2]$ such that

$f(1) < f(c) = 0 < f(2)$. ✓

#16. The statement is FALSE.

Example: $f(x) = |x|$.

$\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{|x|}{x} \neq \lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0}$.