Midterm Exam II Solution Spring 2014 MAT175 Section B401.

Part I

#1. $4x^2+9y^2=25$

Implicit diff. $\downarrow$

$8x\,dx+18y\,dy=0$

(i.e. $8x+18yy'=0$).

$\Leftrightarrow \frac{dy}{dx} = -\frac{8x}{18y} = -\frac{4x}{9y}$

$\left.\frac{dy}{dx}\right|_{(2,1)} = -\frac{4\cdot 2}{9\cdot 1} = \underline{\underline{-\frac{8}{9}}}$ //.

#2. $z=x^3e^{3x}$ Product rule

$\frac{dz}{dx} = 3x^2\cdot e^{3x} + x^3\cdot \underbrace{e^{3x}\cdot 3}_{\text{by chain rule}}$

$= \underline{\underline{3e^{3x}x^2(1+x)}}$ //

#3. $p(l)=\ln(l^2+\sin l)$ chain rule

$p'(l) = \underline{\underline{\frac{1}{l^2+\sin l}\cdot(2l+\cos l)}}$ //

#4. $A(a)=\frac{\sqrt{3}}{4}a^2$. $\frac{dA}{dt}=3$ inch²/sec.

When $A=4\sqrt{3}$, what's $\frac{da}{dt}$?

Sol When $A=4\sqrt{3}$, from $\frac{\sqrt{3}}{4}a^2=4\sqrt{3}$,

$\boxed{a=4}$. Now

$\frac{dA}{dt}=\frac{\sqrt{3}}{2}a\frac{da}{dt}$

When $a=4$, $3=\frac{\sqrt{3}}{2}\cdot 4\cdot\frac{da}{dt}$

$\frac{da}{dt}=\frac{3}{2\sqrt{3}}=\underline{\underline{\frac{\sqrt{3}}{2}}}$ //.

#5. $\lim_{x\to\infty}\frac{2x^2+x+2}{3x^2-x+1} = \lim_{x\to\infty}\frac{2+\frac{1}{x}+\frac{2}{x^2}}{3-\frac{1}{x}+\frac{1}{x^2}} = \underline{\underline{\frac{2}{3}}}$ //.

#6. First note that

$\lim_{t\to\infty}\frac{\pm 1}{1-e^t}=0.$

From $\frac{-1}{1-e^t}\leq\frac{\cos t}{1-e^t}\leq\frac{1}{1-e^t}$,

As $t\to\infty$ $\downarrow$ 0   As $t\to\infty$ $\downarrow$ 0

by Squeeze theorem,

$\lim_{t\to\infty}\frac{\cos t}{1-e^t}=\underline{\underline{0}}$ //

#7.

$\lim_{x\to\infty}\frac{2013x^3+2014x^2+2015x+2016}{2013x^3+2012x^2+2011x+2010}$

$=\lim_{x\to\infty}\frac{2013+2014\frac{1}{x}+2015\frac{1}{x^2}+2016\cdot\frac{1}{x^3}}{2013+2012\frac{1}{x}+2011\frac{1}{x^2}+2010\frac{1}{x^3}}$

$=\frac{2013}{2013}=\underline{\underline{1}}$ //.

#8. $\lim_{x\to\infty}\frac{x+\frac{1}{x}}{x^2-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{\cancel{x+\frac{1}{x}}}{(x-\frac{1}{x})\cancel{(x+\frac{1}{x})}}$

$=\lim_{x\to\infty}\frac{1}{x-\frac{1}{x}} = \underline{\underline{0}}$ //

Part II

#9. $f(x) = 2x^3 - 4x^2$ on $[-1, 2]$.

$f'(x) = 6x^2 - 8x \overset{\text{put}}{=} 0$

$\Leftrightarrow 3x^2 - 4x = 0$

$\Leftrightarrow (3x-4)\cdot x = 0.$

$x = \frac{4}{3}$ or $x = 0$.

From Endpoints	From Critical #s
$f(-1) = -6$	$f(\frac{4}{3}) = 2\cdot(\frac{4}{3})^3 - 4\cdot(\frac{4}{3})^2 = \frac{2\cdot 4^3}{3^3} - \frac{4^3\cdot 3}{9\cdot 3} = -\frac{4^3}{3^3} > -6.$
$f(2) = 0$	$f(0) = 0$

Over $[-1, 2]$,
f has its maximum 0 at $x = 0$ and 2, and its minimum -6 at $x = -1$.

#10. $f(x) = x^3 - x^2$ on $[0, 1]$.

$f'(x) = 3x^2 - 2x \overset{\text{put}}{=} 0.$

$\Leftrightarrow x(3x-2) = 0.$

$\Leftrightarrow x = 0$ or $x = \frac{2}{3}$.

From endpoints:	from Critical #s:
$f(0) = 0$	$f(0) = 0$
$f(1) = 0$	$f(\frac{2}{3}) = (\frac{2}{3})^3 - (\frac{2}{3})^2 = \frac{8}{3^3} - \frac{4\cdot 3}{3^2\cdot 3} = -\frac{4}{27}.$

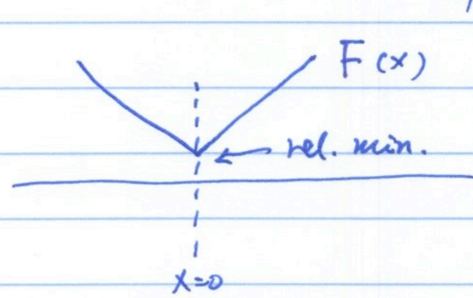
Over $[0, 1]$,
f has its maximum 0 at $x = 0$ and 1, and its minimum $-\frac{4}{27}$ at $x = \frac{2}{3}$.

#11. $F(x) = x^4 + 5x^2 + 6.$

$F'(x) = 4x^3 + 10x \overset{\text{put}}{=} 0.$

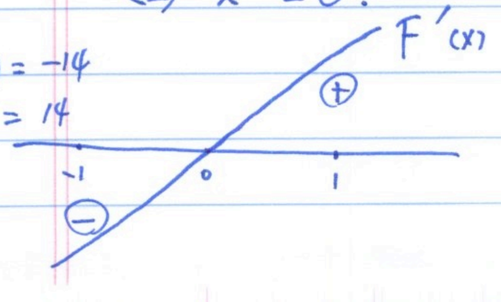
$\Leftrightarrow 2x\underbrace{(2x^2+5)}_{\text{positive for all } x.} = 0$

$\Leftrightarrow x = 0.$



F(x) has relative minimum $F(0) = 6$ at $x = 0$.

$F'(-1) = -14$
$F'(1) = 14$



#12. $F(x) = 2x + \frac{2}{x}$. $x=0$ is a critical number since $F(x)$ is not defined when $x=0$.

$F'(x) = 2 - \frac{2}{x^2} \overset{\text{put}}{=\!=} 0$.

$\Leftrightarrow x^2 - 1 = 0$

$\Leftrightarrow (x-1)(x+1) = 0$

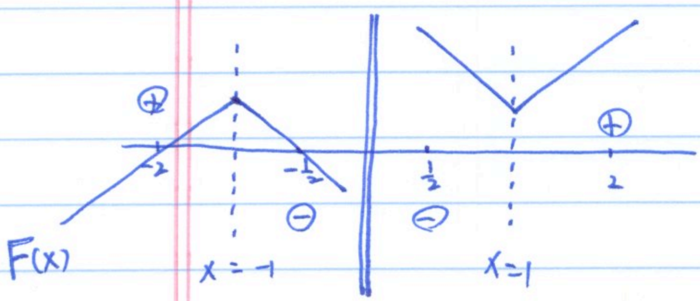
$\Leftrightarrow x = 1$ or $x = -1$.

Testing values $x = -2, -\frac{1}{2}, \frac{1}{2}, 2$

(In fact since $F'(x)$: even function, it suffices to find $F'(\frac{1}{2})$ and $F'(2)$).

$F'(\frac{1}{2}) = 2 - \frac{2}{\frac{1}{4}} = 2 - 8 = -6 = F'(-\frac{1}{2})$

$F'(2) = 2 - \frac{2}{4} = \frac{3}{2} = F'(-2)$.



$F(x)$ has relative maximum at $x = -1$ as $F(-1) = -4$.

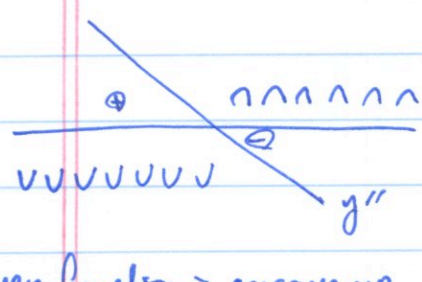
$F(x)$ has relative minimum at $x = 1$ as $F(1) = 4$.

#13. $y = -x^3 + x^2 + 2x - 1$.

$y' = -3x^2 + 2x + 2$

$y'' = -6x + 2 \overset{\text{put}}{=\!=} 0$

$\Leftrightarrow x = \frac{1}{3}$.



$y\big|_{x=\frac{1}{3}} = -\frac{1}{27} + \frac{1}{9} + \frac{2}{3} - 1$

$= \frac{-1 + 3 + 18 - 27}{27}$

$= \frac{-7}{27}$.

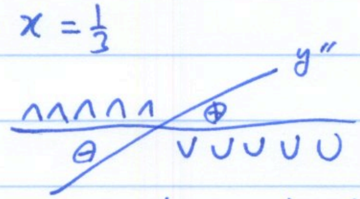
The given function is concave up on $(-\infty, \frac{1}{3})$, Concave down on $(\frac{1}{3}, \infty)$. Inflection pt: $(\frac{1}{3}, -\frac{7}{27})$. : Answer

#14. $y = x^3 - x^2$.

$y' = 3x^2 - 2x$

$y'' = 6x - 2 \overset{\text{put}}{=\!=} 0$.

$\Leftrightarrow x = \frac{1}{3}$



$y\big|_{x=\frac{1}{3}} = \frac{1}{27} - \frac{1}{9} = -\frac{2}{27}$.

Answer: The given function is concave up on $(\frac{1}{3}, \infty)$, Concave down on $(-\infty, \frac{1}{3})$. Inflection pt: $(\frac{1}{3}, -\frac{2}{27})$.