Exercise 2.2.8 (b)

$$\lim_{(x,y)\to(0,0)} \frac{\sin xy}{y} = 0.$$

Proof. First notice that $|\sin\theta| \le |\theta|$ if $|\theta|$ is a very small positive number close to 0. This means

$$\left|\frac{\sin xy}{y}\right| \le \frac{|xy|}{|y|} = |x| \le \sqrt{x^2+y^2}. \tag{1}$$

We now use the $\varepsilon - \delta$ argument to show that the limit is actually 0. We have to show: For every $\varepsilon > 0$, there exists $\delta > 0$ such that $0 < \|(x,y)-(0,0)\| = \sqrt{x^2+y^2} < \delta$ implies $\left|\frac{\sin xy}{y} - 0\right| < \varepsilon$. By the inequality (1) above, we can take $\varepsilon = \delta$. □