

We wanted to solve the following system of equations.

$$\begin{cases} 3x^2 - 6xy + 3y^2 = 12x & \dots \textcircled{1} \\ -3x^2 + 6xy - 3y^2 = 12y & \dots \textcircled{2} \\ x^2 + y^2 = 1 & \dots \textcircled{3} \end{cases}$$

From $\textcircled{1} + \textcircled{2}$

we get $0 = 2\lambda(x+y)$

i.e. Case 1: $\lambda = 0$

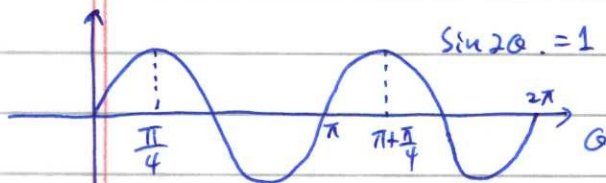
Case 2: $x = -y$.

when $\lambda = 0$, from $\textcircled{1}$ and $\textcircled{3}$

we have $3 - 6xy = 0 \Leftrightarrow xy = \frac{1}{2}$.

Let $x = r \cos \theta$, $y = r \sin \theta$ with $r = 1$. ($\because \textcircled{3}$)

$$\frac{1}{2} = xy = \cos \theta \sin \theta \Leftrightarrow 1 = 2 \cos \theta \sin \theta = \sin 2\theta.$$



$\sin 2\theta = 1$ when $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$.

when $\theta = \frac{\pi}{4}$ $x = \frac{\sqrt{2}}{2}$, $y = \frac{\sqrt{2}}{2}$

when $\theta = \frac{3\pi}{4}$ $x = -\frac{\sqrt{2}}{2}$, $y = \frac{\sqrt{2}}{2}$

which coincide with the result through another approach.