Take-home Final Exam

MATH 250 Section 02

From May 18th, 2016 7:25pm to May 25th, 2016 5:20pm.

Deadline: May 25th, 2016 5:20pm.

Instructions: Policies of this exam are described on coversheet. You can keep this problem sheet. Please submit your solutions with the coversheet stapled on the top. Please note that late-submissions are NOT accepted.

1. Let $f(x,y) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + \frac{1}{3}y^3 - \frac{5}{2}y^2 + 6y - 9$. Find and classify all critical points of f.

2. Extremize $f(x,y,z) = x$ subject to the constraints $x^2 + y^2 + z^2 = 1$ and $x + y + z = 1$.

3. Prove if the following statement is true, or disprove by giving an example if it is false:
Let $f : A \subset \mathbb{R}^n \to \mathbb{R}$ be a continuous function on A whose all first partial derivatives $\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}$ exist at $\overrightarrow{x}_0 \in A$. Then the function f is differentiable at $\overrightarrow{x}_0 \in A$.

4. Show that, at a local maximum or minimum of $\|\overrightarrow{r}(t)\|$, the vector $\overrightarrow{r}'(t)$ is perpendicular to $\overrightarrow{r}(t)$.

5. State and prove Fubini's theorem for the case of continuous functions defined on a rectangle.

6. (1) Let

$$f(m,n) := \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \cos nx \sin my \, dx dy.$$

Calculate $\lim_{m,n\to\infty} f(m,n)$.

(2) Let D be the region bounded by the positive x and y axes and the parabola $y = -x^2 + 1$. Compute

$$\iint_D (x^2 + xy - y^2) dA.$$

Please see overleaf

7. Sketch the region and compute

$$\int_0^4 \int_{y/2}^2 e^{x^2} \, dx dy.$$

8. Let $W := \{(x, y, z) \in \mathbb{R}^3 : \frac{1}{2} \leq z \leq 1 \text{ and } x^2 + y^2 + z^2 \leq 1\}$. Sketch the region and set up a triple integral representing the volume of W. (Do not calculate the integral.)

9. Let $a > 0$. Show that

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi},$$

and compute

$$\int_{-\infty}^{\infty} e^{-ax^2} dx.$$

10. Find the moment of inertia around the y-axis for the ball $\mathbb{B}_R = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq R^2\}$ if the mass density is a constant μ.