Corrigendum to Review Problems for Chapters 1 and 2
Spring 2016 MATH 250 Section 02

Acknowledgement. Many thanks to Dariusz for pointing out errors.

The handout states:

6. (1) Prove if the following statement is true, or disprove by giving an example if it is false:
Let $f : A \subset \mathbb{R}^n \to \mathbb{R}$ be a continuous function on A whose all first partial derivatives $\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}$ exist at $\overrightarrow{x}_0 \in A$. Then the function f is differentiable at $\overrightarrow{x}_0 \in A$.

It should be read:

6. (1) Prove if the following statement is true, or disprove by giving an example if it is false:
Let $f : A \subset \mathbb{R}^n \to \mathbb{R}$ be a continuous function on A whose all first partial derivatives $\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}$ exist at $\overrightarrow{x}_0 \in A$. Then the function f is differentiable at $\overrightarrow{x}_0 \in A$.

Note: The conclusion is about the differentiability at a vector $\overrightarrow{x}_0 \in A$, not the entire domain.

The handout states:

6. (3) Prove if the following statement is true, or disprove by giving an example if it is false:
Let $f : A \subset \mathbb{R}^n \to \mathbb{R}$ be a function on A whose all first partial derivatives $\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}$ exist at $\overrightarrow{x}_0 \in A$. Then the function f is continuous.

It should be read:

6. (3) Prove if the following statement is true, or disprove by giving an example if it is false:
Let $f : A \subset \mathbb{R}^n \to \mathbb{R}$ be a function on A whose all first partial derivatives $\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}$ exist at $\overrightarrow{x}_0 \in A$. Then the function f is continuous at $\overrightarrow{x}_0 \in A$.

Note: The conclusion is about the continuity at a vector $\overrightarrow{x}_0 \in A$, not the entire domain.

The handout states:

9. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be differentiable at $\overrightarrow{x}_0 \in \mathbb{R}^3$. Prove that

$$\frac{|f(\overrightarrow{x}) - f(\overrightarrow{x}_0)|}{\|\overrightarrow{x} - \overrightarrow{x}_0\|}$$

is bounded by a positive constant. (Hint: Use the triangle inequality and the Cauchy-Schwarz inequality)

It should be read:

9. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be differentiable at $\overrightarrow{x}_0 \in \mathbb{R}^3$. Prove that

$$\lim_{\overrightarrow{x} \to \overrightarrow{x}_0} \frac{|f(\overrightarrow{x}) - f(\overrightarrow{x}_0)|}{\|\overrightarrow{x} - \overrightarrow{x}_0\|}$$

is bounded by a positive constant. (Hint: Use the triangle inequality and the Cauchy-Schwarz inequality)

Note: Without the limit as $\overrightarrow{x}$ goes $\overrightarrow{x}_0$, the statement makes sense, but that is a false statement. In fact there is reason that the quotient is bounded by a constant. Actually one can cook up a counterexample easily: Let

$$f(x,y,z) := z\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2} = z\left\|\overrightarrow{x}-\overrightarrow{x}_0\right\|.$$

Notice that

$$\left|f(\overrightarrow{x})-f(\overrightarrow{x}_0)\right| = z\left\|\overrightarrow{x}-\overrightarrow{x}_0\right\|,$$

and hence

$$\frac{\left|f(\overrightarrow{x})-f(\overrightarrow{x}_0)\right|}{\left\|\overrightarrow{x}-\overrightarrow{x}_0\right\|} = z.$$

Clearly this quotient is not bounded.