Review Problems — Chapter 3
Spring 2016 MATH 250 Section 02

Review Problems

1. (1) Prove or disprove the following statement: Let $f : \mathbb{R}^n \to \mathbb{R}$ be any map. Then

$$\frac{\partial^2 f}{\partial x_i \partial x_j} = \frac{\partial^2 f}{\partial x_j \partial x_i}.$$

(2) Prove or disprove the following statement: Every critical point is a local extremum.

2. Let $f(x, y) = x^4 + x^3 + y^2 + y + xy + 3$. Find the second order Taylor approximation of f at $(1, 2)$.

3. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a quadratic form. Construct a symmetric bilinear form using the quadratic form f. (Recall that a **symmetric bilinear form** is a map $g : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ such that

- $g(\overrightarrow{v}, \overrightarrow{w}) = g(\overrightarrow{w}, \overrightarrow{v})$ for every $\overrightarrow{v}, \overrightarrow{w} \in \mathbb{R}^n$.
- $g(a_1\overrightarrow{v}_1 + a_2\overrightarrow{v}_2, \overrightarrow{w})$ for every $\overrightarrow{v}_1, \overrightarrow{v}_2, \overrightarrow{w} \in \mathbb{R}^n$ and $a_1, a_2 \in \mathbb{R}$.

holds.) Note: You should check what you constructed is actually satisfying the definition of a symmetric bilinear form.

4. Classify all critical points of $f(x, y) = x^2 - 3xy + 5x - 2y + 6y^2 + 8$.

5. For given $f(x, y, z) = x^2 + y^2 + z^2 - 2xyz$ find all critical points and determine whether they are local minima, local maxima, saddle points, or none of them. (Hint: Use determinant test for positive definiteness in p.175.)

6. The proof of **Theorem 8** in p.186 of the textbook has a gap; i.e., the proof is incomplete. (1) Explain what is wrong in the proof. (2) Provide a complete the proof. (Hint: See p.206. You can use the implicit function theorem without proof.)

7. Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 - x - y + 1$ on the unit disc $\mathbb{D}^2 = \{(x, y) : x^2 + y^2 \leq 1\}$.

8. Let $f : \mathbb{R}^2 \to \mathbb{R}$, $(x, y) \mapsto x^2 + xy + y^2$, and S the unit circle in $\mathbb{R}^2$. Find the extrema of $f|_S$ by using the bordered Hessian test. (No credit will be given if there is no use of bordered Hessian test.)

9. Find the extrema of $f(x, y, z) = x + y + z$ subject to $x^2 - y^2 = 1$ and $2x + z = 1$.

If you need any help, please feel free to send an email (or multiple emails) to `byungdpark@gmail.com`.