Review Problems for Chapters 4, 5, and 6
Spring 2016 MATH 250 Section 02

Review Problems

1. Show that if the acceleration of an object is always perpendicular to the velocity, then the speed of the object is constant.

2. Find the arc length of $\alpha(t) = (t, t\sin t, t\cos t)$, for $t \in [0, \pi]$.

3. Let f be continuous on $[a, b] \times [c, d]$; for $a < x < b$, $c < y < d$, define
$$F(x, y) = \int_a^x \int_c^y f(u, v)dudv.$$
Show that $\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial^2 F}{\partial y \partial x} = f(x, y)$.

4. Let
$$f(m, n) := \iint_R x^m y^n dxdy,$$
where $R = [0, 1] \times [0, 1]$. Find $\lim_{m,n\to\infty} f(m, n)$.

5. Let D be the region bounded by the positive x and y axes and the line $3x + 4y = 10$. Compute
$$\iint_D (x^2 + y^2)dA.$$

6. Sketch the region and compute
$$\int_{-1}^1 \int_{|y|}^1 (x + y)^2 dxdy.$$

7. Let $W := \{(x, y, z) \in \mathbb{R}^3 : \sqrt{x^2 + y^2} \le z \le 1\}$. Sketch the region and compute the volume.

8. Let $a > 0$. Compute $\int_{-\infty}^{\infty} e^{-ax^2} dx$.

9. Use spherical coordinate to evaluate
$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \frac{\sqrt{x^2 + y^2 + z^2}}{1 + (x^2 + y^2 + z^2)^2} dzdydx.$$

10. Find the average value of e^{-z} over the unit ball in $\mathbb{R}^3$.

If you need any help, please feel free to send an email (or multiple emails) to `byungdpark@gmail.com`.