

Exam I
Spring 2017 MATH 15500 Section 06
March 7th, 2017. 9:10AM–11:00AM

Your name:

Instructions: Please clearly write your name above. This exam is closed-book and closed-note. You cannot use any electronic device in this exam. You are not allowed to talk to other students. Write all details explicitly. Answers without justifications and/or calculation steps may receive no score. Hand-in both the exam sheet and your work on given sheets.

Total 100 points. 10 points each.

1. Let R be the region bounded by the x -axis, y -axis, and the function $y = \cos x$. Find the volume of the solid generated when R is revolved about the x -axis. (Hint: $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$)

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2. Find the arc length of the curve given by the function

$$y = \frac{e^x + e^{-x}}{2}$$

on $[-\ln 2, \ln 2]$ by integrating with respect to x .

3. For the function $y = 8\sqrt{x}$ on $[9, 20]$, find the area of the surface of revolution obtained by revolving the graph about x -axis.

4. Suppose a force of 30N is required to stretch and hold a spring 0.3m from its equilibrium position. How much additional work is required to compress the spring 0.2m if it has already been compressed 0.3m from its equilibrium?

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5. For the function $f(x) = x^3 - 4$, find the slope of the tangent line on the point $(4, 2)$ of f^{-1} .

6. Evaluate the following integral:

$$\int_0^{\frac{\pi}{2}} \frac{1 + \cos x}{x + \sin x} dx.$$

7. Calculate the following integral:

$$\int \frac{1}{25x^2 + 1} dx.$$

8. Evaluate the limit:

$$\lim_{x \rightarrow 0^+} (\csc x)^x$$

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9. Calculate the following integral:

$$\int \frac{\sin x + \tan x}{\cos^2 x} dx.$$

10. Calculate the following integral:

$$\int \ln x \, dx.$$