

Exam II
Spring 2017 MATH 15500 Section 06
April 4th, 2017. 09:00–11:00

Your name:

Instructions: Please clearly write your name above. This exam is closed-book and closed-note. You cannot use any electronic device in this exam. You are not allowed to talk to other students. Write all details explicitly. Answers without justifications and/or calculation steps may receive no score.

Total 100 points. 10 points each unless specified otherwise.

1. Calculate the following integral:

$$\int \sec^3 \theta d\theta$$

Hint: You may use $\int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C$.

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2. Calculate the following integral:

$$\int \cos^2 x \sin^2 x dx$$

3. Prove that the area of an ellipse, whose equation is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is $ab\pi$.

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4. Calculate

$$\int \frac{x^2}{\sqrt{16-x^2}} dx.$$

Hint: You may use $\sin 2x = 2 \sin x \cos x$.

5. Calculate

$$\int \frac{dx}{x^2 - 3x - 4}.$$

6. (5 points each) Let $f(x) = \frac{1}{x^p}$, where $0 < p < \infty$. Discuss the convergence of the definite integral $\int_1^\infty f(x)dx$ in the following cases:

(1) When $0 < p < 1$:

(2) When $p = 1$:

(3) When $p > 1$:

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7. Find the constant k that satisfies the following equation:

$$\int_{-\infty}^{\infty} \frac{k}{4+x^2} dx = 1.$$

8. Find the value that the following infinite sum converges to:

$$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}.$$

9. Show that the following sequence converges and find the limit.

$$a_n = \frac{(-1)^n}{n!}.$$

Here $n! := n \cdot (n - 1) \cdot \dots \cdot 3 \cdot 2 \cdot 1$.

10 (5 points). Evaluate the following geometric series:

$$1 + \frac{e}{\pi} + \frac{e^2}{\pi^2} + \dots + \frac{e^n}{\pi^n} + \dots$$

Please use this space if you need more space.