Final Examination
Spring 2017 MATH 15500 Section 06
May 19th, 2017. 09:00–11:00

Your name:

Instructions: Please clearly write your name above. This exam is closed-book and closed-note. You cannot use any electronic device in this exam. You are not allowed to talk to other students. Write all details explicitly. Answers without justifications and/or calculation steps may receive no score.

Total 100 points. 10 points each unless specified otherwise.

1. (8 points) The graph of $f(x) = 2\sqrt{x}$ on the interval $[1, 3]$ is revolved about the x-axis. What is the area of surface generated?

2. Let R be the region bounded by $y = \ln x$, the x-axis, and the line $x = e$. Find the volume of the solid generated when the region R is revolved about the x-axis.

3. (5 points each) Evaluate or show divergence:

(1)

$$\int_{-\infty}^{\infty} \frac{1}{x^2+9}dx.$$

(2)

$$\int_{0}^{\infty} \frac{e^{2x}}{e^{2x}+1}dx.$$

4. (3 points each) Compute the limit of the sequence or show divergence:
(1)

$$\lim_{k\to\infty} \frac{e^k}{k^2}.$$

(2)

$$\lim_{n\to\infty} \frac{2\sin n^2}{n^3}.$$

(3)

$$\lim_{n\to\infty} \sum_{k=0}^{n} \left(\frac{3}{2}\right)^k.$$

5. Given an infinite series

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n-1}},$$

show that the series is divergent using indicated methods:

(1) (3 points) The comparison test. (You can use $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ is divergent without proof.)

(2) (7 points) The integral test.

6. Determine whether the following series converges:

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^2}.$$

7. Show that the following series is absolutely convergent, convergent, or divergent:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}.$$

8. Write down the degree 4 Taylor polynomial centered at 0:

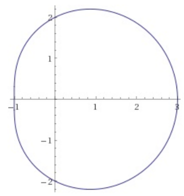
$$p_4(x) = \sum_{k=0}^{4} \frac{f^{(k)}(0)}{k!} x^k$$

for $f(x) = 1 + e^{-x}$.

9. Find the interval of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{n}}.$$

(Verify and clearly mention whether your final answer is a(n) open, half-open, or closed interval!)



10. (1) (3 points) Let C be a circle of radius 2 centered at $(0, 2)$. Write the equation of C in the polar coodinate.

(2) (10 points) Calculate the enclosed area by the limaçon $r = 2 + \cos\theta$ depicted as above.

Please use this space if you need more space.