

Exam II
MATH 155 Section 08
November 5th, 2015. 7:35PM–9:25PM

Your name:

Instructions: Please clearly write your name above. This exam is closed-book and closed-note. You cannot use any electronic device in this exam. You are not allowed to talk to other students. Write all details explicitly. Answers without justifications and/or calculation steps may receive no score. Hand-in this exam sheets and other sheets which contain your work to be graded.

Total 100 points. 10 points each unless specified otherwise.

1. Evaluate the following integral:

$$\int \cos^3 x dx$$

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2. Prove that the area of an ellipse, whose equation is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is $ab\pi$.

3. Evaluate

$$\int \frac{dx}{(1+x^2)^2}.$$

4. Evaluate

$$\int \frac{dx}{x^2 - 5x + 6}.$$

5. Find the constant k that satisfies the following equation:

$$\int_{-\infty}^{\infty} \frac{k}{1 + 4x^2} dx = 1.$$

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6. (5 points each) Let $f(x) = \frac{1}{x^p}$, where $0 < p < \infty$. Discuss the convergence of the definite integral $\int_1^\infty f(x)dx$ in the following cases:

(1) When $0 < p < 1$:

(2) When $p = 1$:

(3) When $p > 1$:

7. Find the value that the following infinite sum converges to:

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)}.$$

8. Find the limit of the sequence:

$$a_n = \frac{\sin n}{n^2 + 1}.$$

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9. Evaluate the following geometric series: $1 + \frac{1}{\pi} + \frac{1}{\pi^2} + \dots + \frac{1}{\pi^n} + \dots$

10. (5 points) Evaluate $\int \ln x dx$. (Hint: Integration by parts.)